A Problem in Geometric Probability:

Buffon’s Needle Problem
The Plan

• Introduction to problem
• Some simple ideas from probability
• Set up the problem
• Find solution
• An approximation
• Generalization (solution known)
• Other generalizations (solutions known?)
Buffon's Needle Problem

Stated in 1733 solution published 1777 by Georges Louis Leclerc, Comte de Buffon (1707-1788)
\[ P(\text{landing on red}) = \frac{\text{red area}}{\text{total area}} \]

\[ P(\text{landing on c}) = \frac{\text{area covered by c}}{\text{total area}} \]
The Set Up

We have a crossing if \( y \leq L \sin \theta \)

\[
d = L \sin \theta \quad 0 \leq \theta \leq \pi \\
0 \leq y \leq D
\]
The Solution!

blue area = \( L \int_0^\pi \sin \theta \, d\theta \)

= \( L(-\cos \pi + \cos 0) \)

= \( 2L \)

total area = \( D\pi \)

\[ p(\text{crossing}) = \frac{\text{blue area}}{\text{total area}} = \frac{2L}{D\pi} \]
Polyá, George (1887, 1985)

… a good teacher should understand and impress on his students the view that no problem whatever is completely exhausted.

Something different

Let $L=1$ and $D=4$, then we have

$$P(\text{crossing}) = \frac{2L}{D\pi} = \frac{2(1)}{4\pi} = \frac{1}{2\pi}$$

We also know that $P(\text{crossing}) \approx \frac{\text{number of actual crossings}}{\text{number of throws}}$

$$\pi \approx \frac{\text{number of throws}}{2 \text{ number of crossings}}$$

Simulation
Generalizations
P(crossing at least on line) = \frac{2L(D+E) - L^2}{\pi DE}

\lim_{E \to \infty} \frac{2L(D+E) - L^2}{\pi DE} = \frac{2L}{\pi D}
Other Generalizations
Any Uses?

$P(\text{crossing}) = \frac{P}{D\pi}$

$P(\text{crossing}) \approx \frac{\text{number of crossings}}{\text{total number of throws}}$

Total number of throws $\approx \frac{D\pi}{P}(\text{number of crossings})$

Counting white blood cells!
function p=bufon(L,D,n)
    cnt=0;
    for i=1:n
        x=rand*(pi/2);
        y=rand*D;
        if y <= (L*sin(x))
            cnt=cnt+1;
        end
    end
    end
    p=cnt/n;
EDU» pi  
ans = 3.14159265

EDU» buffon(1,4,100)  
ans = 2.9412

EDU» buffon(1,4,1000)  
ans = 2.8409

EDU» buffon(1,4,3000)  
ans = 3.1646

EDU» buffon(1,4,10000)  
ans = 3.1586

EDU» buffon(1,4,100000)  
ans = 3.1342