

Roots!

In this problem we explore the positive real roots of a family of polynomials of the form:

$$p_n(x) = x^n - x^{n-1} - \dots - x - 1 \text{ for } n \geq 1$$

Make conjectures for each of the following statements and, of course, provide proofs for each of these conjectures.

1. Find the number positive real roots of p_n .
2. Let r_n be the smallest positive real root of p_n . Find two real numbers a and b with the property that $a \leq r_n \leq b$ for all n .
3. Does the sequence r_n (for $n = 1, 2, \dots$) converge?

Hints:

You will need to brush up on your theorems about roots of polynomials, continuous functions, and bounded sequences. Make sure to include an accurate statement of any theorems that you use in your proofs and carefully explain how you use them.

Feel free to show all the exploration in one section of your paper.