Projectile Motion Gets the Hose

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Students take a weekly quiz in our introductory physics course. During the week in which material focused on projectile motion, we not-so-subtly suggested what problem the students would see on the quiz. The quiz problem was an almost exact replica of a homework problem[1] we worked through in the class preceding the quiz. The goal of the problem is to find the launch speed if the final horizontal and vertical positions and launch angle are given. Figure 1 shows a schematic of the trajectory.

Solving the problem with no air resistance is a straightforward exercise in two-dimensional kinematics. With the launch at the origin of a standard *x-y* Cartesian coordinate system, the final horizontal position x_f is given by

$$x_f = (v_0 \cos\theta_0)\, T, \tag{1}$$

where v_0 is the launch speed, θ_0 is the angle the launch velocity makes with the horizontal, and T is the time of flight. The final position in the vertical direction y_f is given by

$$y_f = (v_0 \sin\theta_0)T - \frac{1}{2} g T^2, \tag{2}$$

where g is the magnitude of the gravitational acceleration ($9.80\ \mathrm{m/s^2}$). Eliminating the time of flight T from Eq. (1) gives

$$y_f = x_f \tan\theta_0 - \frac{g x_f^2}{2 v_0^2 \cos^2\theta_0}, \tag{3}$$

which is easily solved for v_0.

Despite having worked through the above problem in class, the relatively few lines of algebra needed to solve the problem proved quite difficult for our students. We turned to the lab,

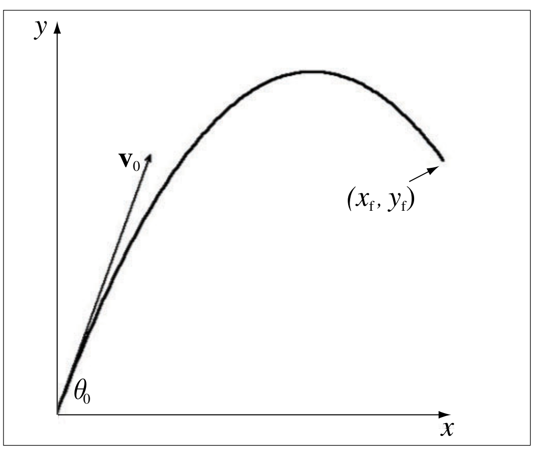

Fig. 1. Parabolic trajectory for a standard introductory projectile motion problem. For the problem of interest here, the launch location (origin) and landing point (x_f, y_f) are given, as well as the angle θ_0 that the launch velocity $\vec{v}_0$ makes with the horizontal. The goal is to find the launch speed v_0.

which met on the two days following the quiz, to help students visualize what challenged them on the quiz. We thought if students could somehow "climb inside" the quiz problem, they could understand the ideas a little better.

The problem we faced is that our standard projectile motion lab does not help students visualize motion. In past years, we have had students either fire a ball out of a launcher or use a ramp to launch a ball bearing off a tabletop. Those labs were done inside our normal introductory physics laboratory, meaning space was limited. The goal of the lab was always to have students use a motion sensor to determine the launch speed, and then do a few calculations to determine the landing point on the floor. They would tape a piece of paper to the floor with a target on it and see how close they came. Our limited space meant that horizontal ranges were rarely more than a meter or two. Those labs also provided little visualization of the trajectory, and students found them frankly a little boring.

Our solution to the lab problem was to go outside and use a water hose. We had seen other ideas[2-5] for using a water hose and thought projected water would help students visualize nearly parabolic motion. A couple of recent papers[6-7] use water to visualize parabolic trajectories. One[6] describes a water drop pulser that allows for parabolic visualization using an indoor instructor's sink. Such a device enhances classroom demonstrations. In the other paper,[7] authors suggest using a cell phone to acquire a snapshot of a water jet; the snapshot is then available for a parabolic fit. Our work here is distinctly different from the two aforementioned papers. We have created a lab that allows students to "climb inside" a parabolic path as large as they are. They take measurements with meter sticks and a protractor without resorting to computational fitting. Also, as we describe shortly, our lab allows instructors to combine two introductory physics topics, namely projectile motion and fluid dynamics, into a single lab experience. In short, our lab has students redo their quiz problem (or homework problem), but this time they "climb inside" the problem and make for themselves the measurements that were given to them on the quiz (or in homework).

Figure 2 shows a water hose clamped to the top of a support rod that stands on a three-pronged base. As seen in that figure, the entire trajectory is visible to students. The nearly parabolic path of the water appears more continuous in real time than it does in Fig. 2 due to the stop-action nature of the photograph. Our students did "climb inside" the parabola in that they walked under the water stream to get a better look at parabolic motion, an action that they told us gave them a better idea of what parabolic motion looks like. We emphasize, though, that it is not necessary to "climb inside" the parabola to make the measurements necessary for completing the lab. We also emphasize that a photograph like the one we show in

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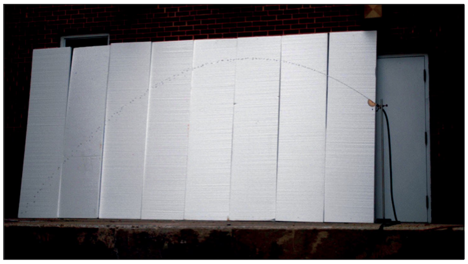

Fig. 2. Our water hose trajectory. We placed white boards behind the water stream so that the stream is more visible. The water leaves the tube 1.30 m above the ground with an initial velocity that is 35° above the horizontal. The water lands 3.66 m horizontally from the base of the support rod.

Fig. 3. The water leaves the glass tube at a launch angle of 35° to the horizontal.

Fig. 2 is not needed to complete the lab; no measurements are taken from a photograph. The outdoor experience of being able to make all measurements on the parabolic water stream is what distinguishes our work from previous work.[7]

Figure 3 shows a close-up view of the end of the hose. From our chemistry storeroom we obtained a #3 rubber stopper, which fit nicely in the end of the hose. The rubber stopper has a premade hole in its center. A 4.6-mm diameter glass tube fit snugly in the stopper's hole. That gave us a thin stream and a launch speed greater than what we would have obtained without the stopper (reducing the cross-sectional area increases the flow speed, a concept we discussed with students). We glued a protractor along the glass tube and used a string and a nut to act as a plumb line.

For the trajectory in Fig. 2, we had students set the origin at the launch point so that it matched what they saw on the quiz. The horizontal range of 3.66 m meant that in the notation used above, $x_f = 3.66$ m. Unlike the trajectory in Fig. 1, the water's landing point is lower than its launch point. That certainly does not change our previous analysis, and we posed questions to the students to make sure they understood that. Because the water landed 1.30 m below its launch point, $y_f = -1.30$ m. With a launch angle of 35° above the horizontal, we are ready to use Eq. (3). Plugging in all the numbers gives $v_0 = 5.03$ m/s, where we keep three significant digits.

The trajectory measurement included errors in the initial height (± 0.5 mm on a meterstick), range (± 10 cm due to droplets breaking up in the middle of the trajectory), and launch angle (± 0.5° on a protractor). Propagating those errors through Eq. (3) gives an uncertainty in the trajectory determination of v_0 to be 0.092 m/s, or about 1.8%. Note of course that air resistance is not part of any of our calculations.

To check our launch speed, we introduce our students to some fluid dynamics. They have not seen this material in class yet, and will not until the end of the first semester. We take our students through the quick derivation[8] of $\rho\, v_0\, A$ being the mass flow rate, where ρ is the water mass density (1000 kg/m^3) and A is the cross-sectional area of the glass tube. We found that water filled a 1.00-L (0.001 m^3) volume glass beaker in 11.93 s, meaning 1.00 kg filled the 1.00-L volume in 11.93 s, giving a flow rate of 0.0838 kg/s. A glass tube diameter of 4.60 mm gives a cross-sectional area of $A = 1.66 \times 10^{-5}$ m^2. With $\rho\, v_0\, A = 0.0838$ kg/s, we get $v_0 = 5.04$ m/s, in excellent agreement with our kinematics result.

The fluid dynamics measurement included errors in timing (± 0.05 s on a stopwatch, including reaction time), on the glass tube diameter (± 0.05 mm on a caliper), and on the water volume (± 10 mL on a granulated beaker). Propagating those errors through gives an uncertainty in the fluid dynamics determination of v_0 to be 0.12 m/s, or about 2.4%.

Aside from students being able to "climb inside" a trajectory, we like our new projectile motion lab for a few other reasons. Students enjoyed leaving their normal indoor laboratory setting and doing something outside. As instructors, we like that our new lab combines two topics from the traditional first semester of introductory physics, namely projectile motion and fluid dynamics. The fact that all of our student groups got almost perfect agreement between the two methods of determining flow speed was an added bonus. The fact that the water stream is not completely continuous also allows us to discuss droplet formation, at least in a qualitative way. Finally, our new projectile motion lab may be used in both our algebra-based and calculus-based introductory courses. It could certainly be used in a high school physics course as well.

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8. See, for example, Chap. 15, p. 249 in Ref. 1.

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