Problem Set #9
Physics 436

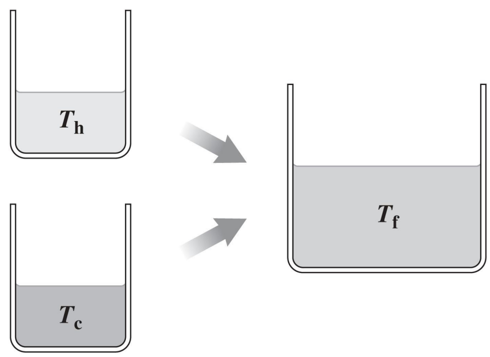
Friday, 25 March 2022

The following two problems come from Wolfson's *Essential University Physics* (1st ed):

- Problem 19.60 on page 325 (*15 points*) ⇒ The problem is given below.

  Find an expression for the entropy change of the whole system in Fig. 19.14, in terms of T_c and T_h, by evaluating the integrals in the paragraph under the heading on Irreversible Heat Transfer on page 319. Assume both glasses contain the same mass, m, of water, whose specific heat c is constant, and neglect the heat capacity of the glasses. Prove that your expression is positive.

  Wolfson's Figure 19.14 is given on page 318 of his book. For completeness, it appears below.

  Do not worry about "the integrals in the paragraph" part. You already know how to evaluate entropy change from what we have done in class. Note that your expression for entropy change involves *only* the temperatures T_c and T_h. Make sure that you *prove* mathematically that the entropy change is positive.

- Problem 19.63 on page 325 (*15 points*) ⇒ The problem is given below.

  You have 50 kg of steam at 100°C, but no heat source to maintain it in that condition. You also have a heat reservoir at 0°C. Suppose you operate a reversible heat engine with this system, so the steam gradually condenses and cools until it reaches 0°C. (a) Calculate the total entropy change of the steam and subsequent water. (b) Calculate the total entropy change of the reservoir. (c) Find the total amount of work that the engine can do. *Hint*: Consider the entropy change of the entire system. This change occurs as the result of a reversible process that supplies useful work. But if it had occurred irreversibly, how much energy would have become unavailable for work?

  This is a nice problem, but it is worded a tad awkwardly. Part (a) is fine and straightforward. For part (b), determine the entropy change of the heat reservoir as if there is no reversible heat engine in the system. In other words, consider just a direct conduction of heat from the steam and subsequent water to the heat reservoir. For part (c), imagine that a reversible heat engine is placed between the 50 kg of steam and the heat reservoir. If the system is reversible, what should the total entropy change of the system be? Once you see the difference between the *irreversible* process of pure heat conduction and the *reversible* process of using a heat engine for work, you can determine how much work the engine can do. You can then calculate how much heat gets dumped into the heat reservoir when the engine is operating. Think about this problem in terms of the "Proposition" I copied for you from the book by Zemansky and Dittman.

The following three problems come from Zemansky and Dittman's *Heat and Thermodynamics* (6th ed):

- Problem 8-7 on page 208 (*15 points*) $\Rightarrow$ The problem is given below.

  (a) A kilogram of water at 273 K is brought into contact with a heat reservoir at 373 K. When the water has reached 373 K, what is the entropy change of the water? Of the heat reservoir? Of the universe?

  (b) If the water had been heated from 273 K by first bringing it in contact with a reservoir at 323 K and then with a reservoir at 373 K, what would have been the entropy change of the universe?

  (c) Explain how the water might be heated from 273 K to 373 K with almost no change of entropy of the universe.

  After offering an explanation in part (c), *prove* mathematically your claim. To do so, you want the entropy change of the universe to be *zero* in an appropriate limit. Simply generalize what you did in part (b). This problem demonstrates the notion of *quasistatic* and the difference between *reversible* and *irreversible*. You might read page 82 of Schroeder's book one more time.

- Problem 8-20 on page 209 (*15 points*) $\Rightarrow$ The problem is given below.

  A body of finite mass is originally at a temperature T_1, which is higher than that of a reservoir at temperature T_2. Suppose that an engine operates in a cycle between the body and the reservoir until it lowers the temperature of the body from T_1 to T_2, thus extracting heat Q from the body. If the engine does work W, it will reject heat $Q-W$ to the reservoir at T_2. Applying the entropy principle, prove that the maximum work obtainable from the engine is

  $$W(\text{max}) = Q - T_2\ (S_1 - S_2)\ ,$$

  where $S_1 - S_2$ is the entropy decrease of the body.

  This problem is essentially the same as Wolfson's Problem 19.63 which appears earlier in the assignment. I think this one is worded a little better. Draw an energy-flow diagram with and without the engine. Show that the energy available for work, which is lost during the irreversible process without the engine, is indeed T_{min} times the entropy change of the universe during the irreversible process. After finishing this short problem, plug the numbers you found from Wolfson's Problem 19.63 into this problem's result to verify what you found in the Wolfson problem. By the way, the "entropy principle" referred to in the problem is simply the fact that "... whenever an irreversible process takes place the entropy of the universe increases" (page 195 in Zemansky and Dittman).

- Problem 8-21 on pages 209-210 (*15 points*) $\Rightarrow$ The problem is given below.

  Two identical bodies of constant heat capacity at temperatures T_1 and T_2, respectively, are used as reservoirs for a heat engine. If the bodies remain at constant pressure and undergo no change of phase, show that the amount of work obtainable is

  $$W = C_p\ (T_1 + T_2 - 2\,T_f)\ ,$$

  where T_f is the final temperature attained by both bodies. Show that, when W is a maximum,

  $$T_f = \sqrt{T_1\,T_2}\,.$$

  This is a very short problem. Use the first law of thermodynamics for the first part and the second law of thermodynamics for the second part.

Due date: **Friday, 01 April 2022**