# Problem Set #4
# Physics 318

17 February 2023

**If you use *Mathematica* or integration tables, reference your work. In the case of using *Mathematica*, include a copy of the code with your work.**

The following problems come from *Introduction to Quantum Mechanics* (2018) by Griffiths and Schroeter:

- Problem 2.21 on page 61 $\Rightarrow$ (*20 points*) You start with an initial wave packet in the shape of a Gaussian. Go through the steps outlined in part (b) because it is a good idea to know how to evaluate this type of integral. Note that there is a slight fudge here because the change of variables involves a *complex* variable. The integration is really done in the complex plane along a line parallel to the real axis; however, the same answer is obtained doing it in the way Griffiths suggests. This is really just luck and you should be careful when doing these types of substitutions in future problems.

- Problem 2.27 on page 70 $\Rightarrow$ (*40 points*) The potential used in this problem is actually a good model of the ammonia molecule. The frequency of the nitrogen atom oscillating between the two potentials can be measured quite accurately and a "clock" is made. Exploit the even symmetry of the potential and use the result of Problem 2.1 (c) to construct the wave functions. I will tell you that if $\alpha \leq \hbar^2/2ma$, there is *one* bound state. If $\alpha > \hbar^2/2ma$, there are *two* bound states. The energies you find will be of the form $E = (\text{some number}) \cdot \hbar^2/ma^2$. Determine what "some number" is for each of the three bound state energies to *three* significant digits (you will have to solve a transcendental equation numerically using, for example, *Mathematica*). I then want you to find the *normalized* wave function for each bound state energy and make a nice computer plot showing all three wave functions together. Use as dimensionless variables, $\tilde{x} = x/a$ for position and $\tilde{\psi} = \psi \cdot \sqrt{a}$ for wave functions. Plot $\tilde{x}$ on the range $-6 \leq \tilde{x} \leq 6$. Your vertical range should be $-1 \leq \tilde{\psi} \leq 1$. If your wave functions exceed this range, or look too small on this range, you have made a mistake.

Due date: **24 February 2023** (*beginning of class*)