

Problem Set #7

Physics 302

Thursday, 21 March 2024

The following problem comes from *Classical Dynamics of Particles and Systems*, Fifth Edition (2004), by Stephen T. Thornton and Jerry B. Marion.

- Problem 4-22 on page 181 \Rightarrow (70 points) A circuit with a nonlinear inductor can be modeled by the first-order differential equations

$$\begin{aligned}\frac{dx}{dt} &= y \\ \frac{dy}{dt} &= -ky - x^3 + B \cos t.\end{aligned}$$

Chaotic oscillations for this situation have been extensively studied. Use a computer to construct the Poincaré section plot for the case $k = 0.1$ and $9.8 \leq B \leq 13.4$. Describe the map.

Instead of hunting around in the dark for what initial conditions and values of B to study, I'll tell you exactly what to use. Marion and Thornton indicate that there are transition regions for $9.8 < B_1 < 9.9$, $11.6 < B_2 < 11.7$, and $13.3 < B_3 < 13.4$. It is not necessary to find B_1 , B_2 and B_3 to great precision. Here are the regions of interest:

- | | | |
|-----------------|-------------------|---|
| $B < B_1$ | \longrightarrow | Period three motion. |
| $B_1 < B < B_2$ | \longrightarrow | Chaos. |
| $B_2 < B < B_3$ | \longrightarrow | Either period one motion or chaos, depending on initial conditions. |
| $B_3 < B$ | \longrightarrow | Period one motion. |

Make the following five graphs:

- Region 1: Phase plot with initial conditions (0,0) and $B = 9.8$. Put the Poincaré section plot on the *same graph* as the phase plot. Note that the latter will just be three points.
- Region 2: Poincaré section plot with initial conditions (0,0) and $B = 10$. Use 8000 drive cycles.
- Region 3: Phase plot with initial conditions (0,0) and $B = 12$. Put the Poincaré section plot on the *same graph* as the phase plot. Note that the latter will just be one point.
- Region 3: Poincaré section plot with initial conditions (10,0) and $B = 12$. Use 8000 drive cycles.
- Region 4: Phase plot with initial conditions (0,0) and $B = 13.4$. Put the Poincaré section plot on the *same graph* as the phase plot. Note that the latter will just be one point.

So that transients are gone, begin each of your plots after 100 drive cycles. Because there are no units given in the problem statement, note that a drive cycle advances t by 2π .

Due date: **Tuesday, 02 April 2024** (*beginning of class*)