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Predicting Winning Times for Stages of the 2011 Tour de France Using an Inclined-Plane Model

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Abstract

Using a modified version of the inclined-plane model I developed to predict winning times for each stage of the Tour de France in the years 2003-05, I present the results of my predicted winning stage times for the 2011 Tour de France. The model incorporates stage profiles, cyclist power input, air drag, and rolling friction. Each stage's predicted winning time was put on my blog the day before a given stage was run. Just one stage prediction was worse than 8% off the actual winning time. Six of the 21 stages were predicted to better than 1% of the actual winning times. The sum of predicted stage-winning times missed the actual sum by 0.5%.

Keywords: Tour de France; cycling; bicycle; modelling

1. Introduction

I began modeling the Tour de France with Benjamin Lee Hannas in 2003. We applied an inclined-plane model to stage-profile data and generated predictions of winning times for each stage of the three-week race. Our idea was to make use of previously published work on biker power input, cycling aerodynamics, and tire-road friction to predict the winning time of each Tour de France stage before a given stage was run. We were not trying to predict the time for any particular cyclist; we wanted to predict the time using a model cyclist competing near the apex of athletic achievement. Our seminal work [1-2] in this area proved quite successful as we were able to predict the majority of stage-winning times for the 2003 and 2004 race to better than 10%. We modeled the 2005 race, but noticed that our early stage predictions were slow because of strong tailwinds that permeated the first week of the race. Only

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after accounting for wind could we bring our early stage predictions in line with the actual results. Because our goal was to predict winning times before each stage was run, we did not publish our 2005 results.

Modeling a race like the Tour de France may at first appear to be almost impossible. There are more than 150 competitors, hairpin turns, peloton dynamics, time trials, varying weather conditions, cars on the road, crashes, and many other aspects of the great race that seem difficult to include in any model. Riders also employ strategies like drafting and breaking away from pelotons that are not known to modelers, eat while cycling, and even take restroom breaks while riding. Making a successful model of the Tour de France thus requires a balance between having enough detail to account for terrain changes and forces on a bicycle, but not so much detail that a crash or food break can significantly worsen a model's predictions. The hope is that a good choice of parameters will be made ahead of time, and that all the aspects of the race that seem unreachable via model will mostly average themselves out.

I returned to Tour de France modeling in 2011. Instead of waiting for my predictions of the stage-winning times to come out in a publication well after the race had finished, I posted my predictions on my blog [3]. What follows in this paper is a description of the inclined-plane model I used to model the 2011 Tour de France.

2. Inclined-Plane Model

The Tour de France website [4] publishes stage-profile data for the 21 stages of the Tour de France. Those profiles appear in June, not too long before the start of the race. Fig 1 shows one of the profiles from the 2011 Tour de France, that of stage 16. The horizontal axis shows distances in kilometers. Those are the distances actually traversed by the cyclists, not the horizontal components of the cyclists' displacements. The numbers in meters show the elevations above sea level at various points along the stage. Note that 11 profile data points are given in Fig 1. I use those points to create a sequence of 10 inclined planes. For each inclined plane, the net distance biked is the hypotenuse, and the net change in elevation is the vertical distance the plane rises. For example, I form an inclined plane between Nyons and Sahune from Fig 1. The hypotenuse of the right triangle formed by the inclined plane is (51 km – 36 km) = 15 km. The vertical height of the right triangle is (355 m – 293 m) = 62 m. Entering in all of the profile data points thus allows me to create a sequence of inclined planes for the entire race.

The physics of a single inclined-plane is all that is required. Once the equation of motion is solved for one inclined-plane, the procedure may be repeated for as many inclined planes as desired.

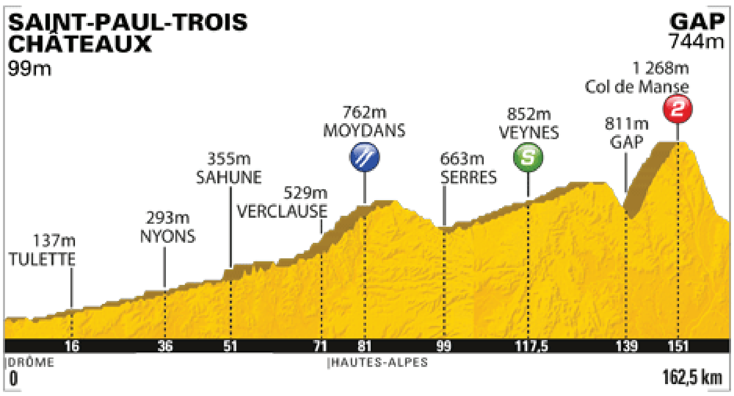

Fig. 1. Profile of stage 16 of the 2011 Tour de France [4]

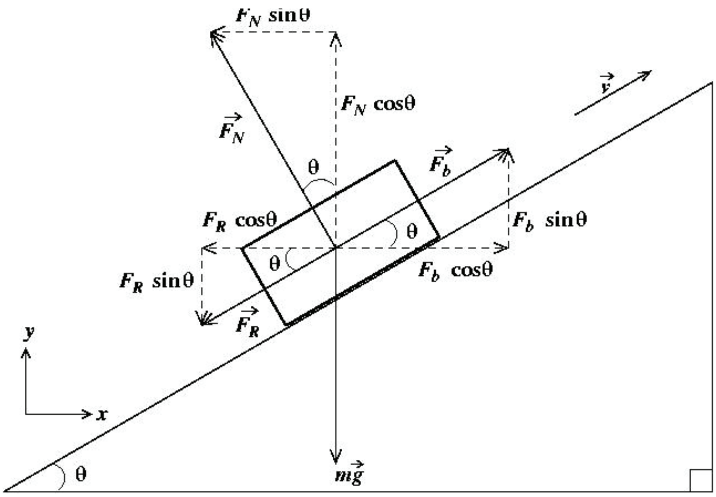

Fig. 2. Free-body diagram of rider-bike combination on an inclined-plane

Fig 2 shows a free-body diagram of a bike-rider combination on an inclined plane. At the instant shown, the bike's velocity points up the plane. Also shown are the forces acting on the bike-rider combination. The magnitude of the combined weight of the bike and rider is mg, where m = 77 kg is the mass of the bike-rider combination and g = 9.80 m/s^2 is the magnitude of the acceleration due to gravity. The force $\vec{F}_b$ is from the road pushing forward on the tires because the tires push back on the road. Though the force is strictly from the road, it comes about because the cyclist inputs power into the pedaling motion, which is responsible for the bike's tires pushing back on the road. The magnitude of that force is $F_b = P_b/v$, where P_b is the biker's power input and v is the bike's center-of-mass speed with respect to the stationary ground. To keep that force physically realistic at small speeds, I assume that for $v < 6$ m/s, the force F_b is constant and given by $P_b/(6\text{ m/s})$ [5].

There are two retarding forces that contribute to $\vec{F}_R$, both of which point opposite the bike's velocity. One is due to the drag force of air resistance, which is given by $F_D = ½\ C_D\rho A v^2$, where C_D is the dimensionless drag coefficient, ρ = 1.2 kg/m^3 is the air density, and the cross-sectional area of the bike-rider combination is A. The other retarding force is due to the rolling friction between the tires and the road. Its magnitude is given by $F_r = \mu_r F_N$, where μ_r is the dimensionless coefficient of rolling friction and F_N is the magnitude of the normal force on the bike-rider combination.

Fig 2 shows that all forces on the bike-rider combination have been split into horizontal and vertical components. The hypotenuse length and vertical height, both of which have already been described as coming from the profile data, determine the inclined plane's angle θ. Thus for any given stage, all stage data are converted into inclined planes with the forces seen in Fig 2. Starting each stage from rest, a cyclist's motion is then determined by the numerical solution of the equation of motion given in Newton's second law.

3. Parameter Choices

Much research has been done on biker power input, drag on bike-rider combinations, and rolling resistance. See, for example, the book [6] edited by Edmund R. Burke, in which there are many articles containing measured values of parameters I need for my model. For the coefficient of rolling resistance, I take μ_r = 0.003. The drag coefficient is difficult to measure because of the varying cross-sectional area of the rider-bike combination. Instead, the product $C_D A$ is often measured. To keep from getting too complicated, I take two possible values of $C_D A$, one for uphill ($\theta > 0$) motion in which $C_D A = 0.35\ \mathrm{m}^2$, and the other for downhill ($\theta \leq 0$) motion in which $C_D A = 0.25\ \mathrm{m}^2$.

The cyclist's power input greatly affects stage-completion time. Going up steep hills requires cyclists to input an enormous amount of power. Fast downhills do not require nearly as much power input. In fact, increasing power on a fast downhill does not contribute much to the speed because the cyclist is at or near terminal speed. For a given power input, terminal speeds are reached rather quickly, regardless of the value of θ. By examining every angle in all 21 stages of the Tour de France, I look for gaps where I can vary the power. I split the power dependence on angle into four regions. The following equation describes how the power is varied with angle.

$$P_b = \begin{cases} 200\ \mathrm{W}, & \theta < -0.05\ \mathrm{rad} \\ P_{main}, & -0.05\ \mathrm{rad} < \theta < 0.06\ \mathrm{rad} \\ P_{upper}, & 0.06\ \mathrm{rad} < \theta < 0.09\ \mathrm{rad} \\ 500\ \mathrm{W}, & 0.09\ \mathrm{rad} < \theta \end{cases} \tag{1}$$

The values of P_{main} and P_{upper} vary depending on the stage. Table 1 includes the values of P_{main} and P_{upper} that I used for the 2011 Tour de France. To determine those power numbers, I used my previous work [1-2] and my subjective estimates of how I thought the cyclists would be affected by fatigue as they moved from one stage to the next. The largest power of 500 W, which could only be sustained by a biker for a short time, is used on only about five or fewer angles for a given Tour de France race. The Tour de France website [4] provides a total of 261 possible inclined-planes for the 2011 race. I added 38 more from Google Earth to fill in some problematic gaps in the online profiles. There are thus 299 total inclined planes for the 21 stages in my 2011 Tour de France model. Of those 299 inclined planes, just three are steep enough to require 500 W. There are 15 inclined planes that require 200 W, and 92 that need P_{upper}. The majority of the inclined planes, 189 of them or about 63% of the total, use P_{main}. Previous models [1-2] did not vary P_{main} and P_{upper} so much. With the 2011 race, I varied them more based on what type of stage was being run, what was run the day before, and where the stage fit into the entire race. For example, the final stage is mostly ceremonial, and I turned down the power considerably.

The 2011 Tour de France had two time trials. Stage 2 was a team time trial; stage 20 was an individual time trial. To account for more aerodynamic racing during a time trail, I reduced the size of $C_D A$ by 20% for those two stages. I also set $P_{main} = P_{upper} = 475$ W to account for the ability of a cyclist to output a large amount of energy in a short amount of time. Stage 2 was won in less than half an hour; stage 20 was won in less than an hour.

4. Model Results

Table 1 shows the results of my model. Except for the team time trial in stage 2, I kept the power splits the same up until stage 12. I dropped P_{upper} a little for stages 12-14 because they were mountain stages. The reduction in power appears to have worked well. For stage 15, a flat stage, I thought there would be fatigue following three grueling mountain stages. I should have kept P_{main} at 325 W instead of lowering it to 300 W.

Stage 16 is by far the most enigmatic for me. That stage followed a rest day. It was also mostly uphill, as Fig 1 shows. Cyclists had to contend with rain for much of that stage. Despite the weather conditions and the mostly uphill nature of the stage, Thor Hushovd won the stage in just over half an hour faster than my prediction. It was the only stage in which I missed the actual time by more than 8%. What is still a mystery to me is how Hushovd finished the stage with an average speed of 12.80 m/s (28.6 mph), which was faster than the winner's average speed for every other stage with the exception of the team time trial in stage 2. Tony Martin won the individual time trial in stage 20 with an average speed of 12.75 m/s (28.5 mph), just a little slower than Hushovd's average speed on stage 16.

Table 1. Model and actual results for 2011 Tour de France

Stage	P_{main} (W)	P_{upper} (W)	Actual	Predicted	Difference	% Difference
1	325	425	4h 41' 31"	4h 40' 01"	-01' 30"	-0.53
2	475	475	0h 25' 16"	0h 26' 35"	01' 19"	5.21
3	325	425	4h 40' 21"	4h 44' 55"	04' 34"	1.63
4	325	425	4h 11' 39"	4h 09' 29"	-02' 10"	-0.86
5	325	425	3h 38' 32"	3h 52' 05"	13' 33"	6.20
6	325	425	5h 13' 37"	5h 20' 13"	06' 36"	2.10
7	325	425	5h 38' 53"	5h 11' 43"	-27' 10"	-8.02
8	325	425	4h 36' 46"	4h 49' 26"	12' 40"	4.58
9	325	425	5h 27' 09"	5h 19' 43"	-07' 26"	-2.27
10	325	425	3h 31' 21"	3h 41' 08"	09' 47"	4.63
11	325	425	3h 46' 07"	3h 46' 05"	-00' 02"	-0.01
12	325	400	6h 01' 15"	5h 59' 26"	-01' 49"	-0.50
13	325	400	3h 47' 36"	3h 46' 52"	-00' 44"	-0.32
14	325	400	5h 13' 25"	5h 02' 45"	-10' 40"	-3.40
15	300	425	4h 20' 24"	4h 36' 38"	16' 14"	6.23
16	350	400	3h 31' 38"	4h 05' 59"	34' 21"	16.23
17	325	400	4h 18' 50"	4h 32' 07"	13' 17"	5.13
18	325	425	6h 07' 56"	5h 51' 23"	-16' 33"	-4.50
19	325	412.5	3h 13' 25"	2h 57' 54"	-15' 31"	-8.02
20	475	475	0h 55' 33"	0h 51' 06"	-04' 27"	-8.01
21	225	225	2h 27' 02"	2h 27' 40"	00' 38"	0.43
TOTAL			85h 48' 16"	86h 13' 13"	24' 57"	0.48

Stages 17-19 were mountain stages. In hindsight, I should have reversed the value of P_{upper} for stages 17 and 18. My thinking ahead of time was that the race would be harder fought near the end of the race, thus making the athletes work a little harder. For stage 19, I averaged the two previous values of P_{upper}, and it turned out to be a bit too much power.

Note that the total of the actual times corresponds to the sum of the winning times for all 21 stages. It does not correspond to Cadel Evans's winning time of 86h 12' 22". That winning time turned out to be just 51 seconds off from the sum of my predicted times. But, as I noted earlier, my goal was not to predict the winning time for any particular athlete; I tried to predict the winning time for each stage. Except for the enigmatic stage 16, I predicted the winning time to better than about 8% for all stages. I predicted the winning time of six stages to better than 1%. The overall error of just under 0.5% does arise because of some fortuitous canceling; i.e. some stages were a bit fast whereas others were a bit slow. Adding all errors in quadrature gives an overall error of 1.16%, which suggests to me a successful model.

I will most likely model the 2012 Tour de France and post predictions on my blog [3]. More details about modeling the Tour de France may be found in my work with Benjamin Hannas [1-2] and in Chapter 4 of my book [7].

Acknowledgements

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