INSTITUTE OF PHYSICS PUBLISHING EUROPEAN JOURNAL OF PHYSICS

Eur. J. Phys. **26** (2005) 251–259 doi:10.1088/0143-0807/26/2/004

Inclined-plane model of the 2004 Tour de France

Benjamin Lee Hannas[1] **and John Eric Goff**[2]

[1] Department of Mathematics, North Carolina State University, Raleigh, NC, 27695, USA
[2] School of Sciences, Lynchburg College, Lynchburg, VA 24501, USA

E-mail: blhannas@unity.ncsu.edu and goff@lynchburg.edu

Received 29 August 2004, in final form 1 November 2004
Published 27 January 2005
Online at stacks.iop.org/EJP/26/251

Abstract

We make modifications to a model we created to predict the stage-winning times of the 2003 Tour de France. With the modifications in place, we model the 2004 Tour de France. We demonstrate here the utility of our 2003 model with its successful application to the 2004 race; our total of all stage-winning times is just 0.05% different from the actual total. We also apply our 2004 model to the 2003 Tour de France and compare our results with our 2003 model. Our 2004 model misses the actual sum of stage-winning times in the 2003 race by just 1.77%.

1. Introduction

We recently published an article [1] in the *American Journal of Physics* that described our model of the 2003 Tour de France. We believe our follow-up work is well suited for an audience more dominated by scientists in Europe, where cycling is a much more popular sport than in the US. Our goal in that first paper was to predict the winning time for each of the 21 stages (including stage 0, or the 'Prolog'). Our sum of predicted stage-winning times differed from the actual total by just 0.03%, though the relative error in quadrature was 1.54%. While some fortuitous cancellation of larger individual stage errors led to such a small overall discrepancy, we feel our model established a novel approach for predicting stage-winning times of Tour de France races.

In this paper, we intend to establish the utility of our model by applying it to the 2004 Tour de France. At the time of this writing, Lance Armstrong had just won his record-breaking sixth straight Tour de France. We did not set out to predict Armstrong's individual stage times, but rather the winning time for each stage. We modified our previous model parameters to account for this year's race having a few more steep mountains[3]. We also relied on more

[3] The Tour de France Web site is www.letour.fr. (We used the English version.) The detailed stage profiles were only available about a week before the race began.

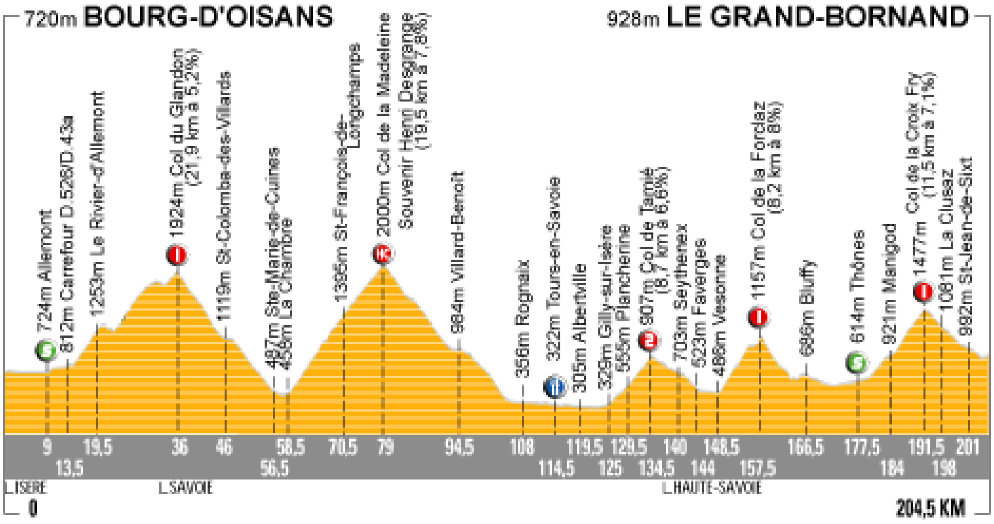

Figure 1. Dynamic profile of stage 17 of the 2004 Tour de France. Image courtesy of Amaury Sport Organisation.

recent publications to provide us with better cyclist power estimates. Thus, in this work, we use our original geometrical model of the course terrain while we adjust the model parameters to better describe the 2004 race.

This paper is organized as follows. Section 2 provides an overview of our previously published Tour de France model. In section 3, we discuss our parameter modifications. In section 4, we present the computational results of our model and discuss our results.

2. Model overview

For a much more complete description of our model than what follows, see our original paper [1]. For completeness, we give a brief overview here. We began our modelling with the stage profiles given in the Tour de France Web site (see footnote 3). Figure 1 shows a typical stage profile, that of stage 17. The distances given on the horizontal are the actual distances biked in kilometres; they are not the horizontal projections of the biked distances. The numbers given in metres above the terrain are the elevations above sea level. To model the terrain, we simply take each pair of distances for a given point and create an inclined plane. The distance biked represents the hypotenuse while the elevation number represents the incline's height. For the stage described in figure 1, we created 27 inclined planes.

Our terrain model obviously does not account for three-dimensional aspects, such as winding turns. However, if enough data points are given on the stage profile, a reasonable model of the terrain using a sequence of inclined planes is achieved. We caution the reader to study the profiles very carefully[4].

The rider–bicycle combination of mass m moves from one inclined plane to another. In addition to the gravitational ($m\vec{g}$, where $\vec{g}$ is the acceleration due to gravity) and normal ($\vec{F}_N$ with magnitude F_N) forces acting, the mass is subjected to two frictional forces and one

[4] One must often magnify the stage profile images to discern some of the numbers. For stage 8, the sixth, seventh and eighth elevations given appear to contradict the terrain image. We chose to use the numbers in that case and did not try to estimate what the terrain image would give.

forward force due to the cyclist's power input. The frictional forces arise from air resistance [2] and rolling friction [3] and are given by

$$F_D = \tfrac{1}{2} C_D \rho A v^2 \quad (1)$$

and

$$F_r = \mu_r F_N, \quad (2)$$

respectively. Here, C_D is the dimensionless drag coefficient, ρ is the density of air, A is the frontal cross-sectional area of the rider–bicycle combination, v is the bicycle's speed and μ_r is the dimensionless coefficient of rolling friction. Both frictional forces are directed opposite to the bike's velocity.

The forward force, $\vec{F}_b$, from the cyclist's power input, P_b, is directed parallel to the bike's velocity and has a magnitude given by

$$F_b = P_b / v. \quad (3)$$

We use a scheme from Giordano [4][5] to handle the apparent divergent nature of equation (3) for small speeds.

With a choice of parameters discussed in the following section, our cyclist begins a stage with zero initial speed. We then simply apply Newton's second law and numerically evaluate the equations of motion for each inclined plane in each stage. Summing up the times needed to traverse all inclined planes in a given stage gives us our prediction of the stage-winning time.

3. Model parameters

The coupling of a good physical model with parameters consistent with reality is essential. There are six parameters at our disposal (excluding our low-speed cutoff for equation (3) [4] (see footnote 5)): m, C_D, ρ, A, μ_r and P_b. We reduce the parameter list by one by taking $C_D A$ as a single parameter. Three parameters are the same in our 2003 and 2004 Tour de France modelling; they are $m = 77$ kg, $\rho = 1.2$ kg m^{-3} and $\mu_r = 0.003$. We refer the reader to our first paper [1] for several references that motivated our parameter choice for the 2003 Tour de France.

For the 2003 race, we took $C_D A = 0.25$ m^2 for downhill motion and $C_D A = 0.35$ m^2 for uphill motion [5][6]. By 'downhill' ('uphill') we simply mean $\theta \leqslant 0$ ($\theta > 0$) where θ is the angle measured from the horizontal for a given inclined plane. We noticed that our 2003 model predicted times for the four short time trial stages (stages 0, 4, 12 and 19 in the 2003 race) that were all too long [1]. For the 2004 race, we modified our values of $C_D A$ to account for the fact that during the time trials, the cyclists wore more aerodynamic clothing and helmets. The riders also used modified bicycles and made great use of drafting[7]. The effect of these efforts was to reduce air drag. We thus modelled three[8] short stages (0, 4 and

[5] The idea is to assume that below some speed, $\tilde{v}$, the cyclist only puts a constant force on the bike. Thus, for $v < \tilde{v}$, $F_b = P_b/\tilde{v}$ while for $v \geqslant \tilde{v}$, equation (3) is used. Giordano uses $\tilde{v} = 7$ m s^{-1}; we use $\tilde{v} = 6$ m s^{-1} believing that a Tour de France cyclist is capable of more force than Giordano assumes.

[6] One nice reference that we missed is [5]. Our values of μ_r and $C_D A$ are consistent with the experimental work in this reference.

[7] 'Drafting' occurs when one cyclist rides immediately behind another, thus reducing air drag. During a team trial, a team will typically ride in single file and continually alternate the leader so that drafting is shared.

[8] We did not reduce $C_D A$ for stage 16, despite the fact that it was a short time trial. Stage 16 began at Bourg d'Oisans and finished at the ski station at L'Alpe d'Huez (see footnote 3). This famous hairpin turn-laden climb through the French Alps covered just 15.5 km of biking distance; however, the cyclists finished 1.13 km higher in elevation compared to where they started (see footnote 3). Suffice to say, speeds were not large enough on this stage to gain much from drafting. Also, most cyclists did not even wear helmets in this stage.

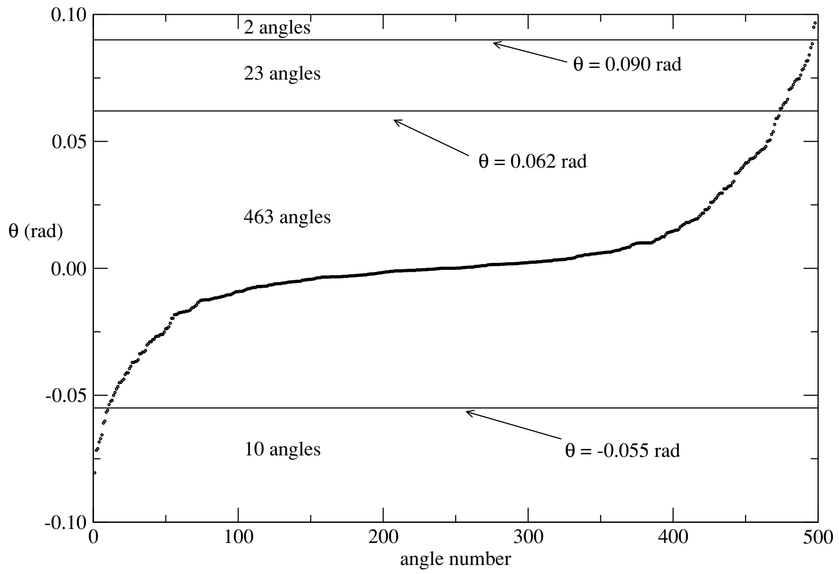

Figure 2. All 498 angles of the 2004 Tour de France shown from left to right in ascending order.

19 in the 2004 race) by reducing our 2003 values of $C_D A$ by 20%, an amount we chose as a reasonable estimate. For all other stages, we kept our 2003 values of $C_D A$.

We discovered [1] in 2003 that our model is most sensitive to the cyclist's power input, P_b. For the 2003 race, we chose

$$P_b = \begin{cases} 200\ \text{W} & (\theta \leqslant -0.055) \\ 375\ \text{W} & (-0.055 \leqslant \theta < 0.09) \\ 500\ \text{W} & (0.09 \leqslant \theta), \end{cases} \tag{4}$$

where the inclined plane angle, θ, is measured in radians. It turned out that 500 W was used for just one of the 401 total number of inclined planes in the 2003 race. The choices of angle cutoff were motivated by examining all angles and looking for separations. Most (385 angles) of the 2003 model used the intermediate power. We note that our model is not very sensitive to the choice of cutoff or power value at the low end. Adding more power on a steep downhill gains very little due to the v^2 dependence in the air drag (see equation (1)).

We determined a total of 498 angles for the 2004 Tour de France, despite having a total distance that was 30 km less than the 2003 race (see footnote 3). Thus, our terrain mesh was a little finer for our 2004 model compared to our 2003 model. Figure 2 shows all 498 angles in an ascending order for the 2004 Tour de France. There are more large angles in the 2004 race compared to the 2003 race. To better account for the increased number of steep inclined planes, we added an extra partition to our power expression. Our 2004 model estimates the following for the cyclist's power outputs on the long stages (excludes stages 0, 4, 16 and 19)

$$P_b = \begin{cases} 200\ \text{W} & (\theta \leqslant -0.055) \\ 325\ \text{W} & (-0.055 \leqslant \theta < 0.062) \\ 425\ \text{W} & (0.062 \leqslant \theta < 0.09) \\ 500\ \text{W} & (0.09 \leqslant \theta). \end{cases} \tag{5}$$

We have simply taken our 2003 model's 375 W value and reduced it by 50 W for flatter segments and increased it by 50 W for the steeper segments. The power choices were

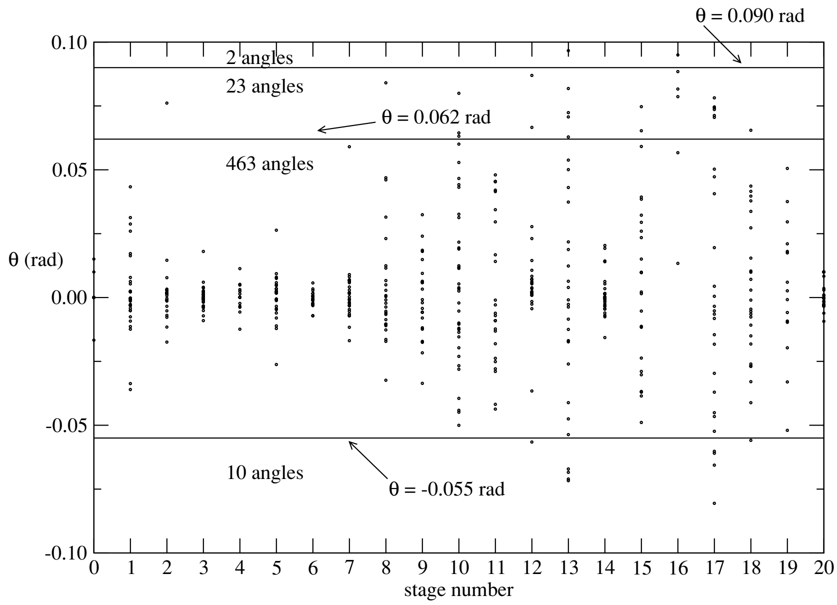

Figure 3. A different look at the 498 angles of the 2004 Tour de France. Note that stages with angles clustered near $\theta = 0$ are fairly flat while stages with a large spread of angles are mountainous.

motivated by a belief that for the 2003 race we overestimated the power on the flat portions of long stages and underestimated the power on the inclined planes at the high end of our intermediate power range. We also feel our current power estimates are more consistent with recent research [6–13][9–13]. As previously mentioned, our model is not sensitive to the low power cutoff or the low power value; hence, we simply kept the same values we used in 2003.

For the short stages 0, 4, 16 and 19, we addressed our 2003 problem of predicting too long a stage time for the time trials by increasing the power on the four short stages. In addition to having more aerodynamic attire and bikes (as discussed at the beginning of this section), a cyclist is capable of outputting larger amounts of power over shorter distances and times. For the time trials, we estimated the power to be

$$P_b = \begin{cases} 200\ \mathrm{W} & (\theta \leqslant -0.055) \\ 475\ \mathrm{W} & (-0.055 \leqslant \theta < 0.09) \\ 500\ \mathrm{W} & (0.09 \leqslant \theta); \end{cases} \tag{6}$$

thus, there is just one intermediate power range. We are again motivated by the literature [6, 8, 11] to believe that our choice of power is reasonable for a Tour de France cyclist.

A different view of the angle data and our choice of angle cutoffs is given in figure 3. One can easily see in figure 3 which stages are affected by our angle cutoffs. Note in particular that

[9] In addition to power values similar to what we use, the work [6] also states drag coefficients consistent with ours.
[10] We note that a Tour de France cyclist is most certainly not in a recumbent body position. However, the paper [7] makes power comparisons with cyclists in a 'standard cycling position' and those power numbers are consistent with what we use here.
[11] [10] is a nice review article with an exhaustive list of references.
[12] Though the experimental tests of [11] were conducted over much shorter time scales than a Tour de France stage, powers averaging around 475 W were achieved by professional cyclists.
[13] The nontechnical paper [13] did not provide us with assistance for parameter selection. We include it, however, because it contains some nice qualitative discussions of rolling resistance and power output.

Table 1. Numerical results for the 2004 Tour de France. The difference column is the difference between column 3 and column 2. The % difference is [(column 3 − column 2)/(column 2)] × 100%. The actual winning times are taken from Web site given in footnote 3.

Stage	Actual winning time	Predicted time	Difference	% difference
0	0 h 06′ 50″	0 h 06′ 51″	00′ 01″	0.24
1	4 h 40′ 29″	4 h 47′ 26″	06′ 57″	2.48
2	4 h 18′ 39″	4 h 38′ 33″	19′ 54″	7.69
3	4 h 36′ 45″	4 h 50′ 57″	14′ 12″	5.13
4	1 h 12′ 03″	1 h 14′ 49″	02′ 46″	3.84
5	5 h 05′ 58″	4 h 46′ 22″	−19′ 36″	−6.41
6	4 h 33′ 41″	4 h 29′ 23″	−04′ 18″	−1.57
7	4 h 31′ 34″	4 h 49′ 25″	17′ 51″	6.57
8	3 h 54′ 22″	3 h 56′ 28″	02′ 06″	0.90
9	3 h 32′ 55″	3 h 49′ 54″	16′ 59″	7.98
10	6 h 00′ 24″	6 h 00′ 01″	−00′ 23″	−0.11
11	3 h 54′ 58″	3 h 54′ 59″	00′ 01″	0.01
12	5 h 03′ 58″	5 h 29′ 24″	25′ 26″	8.37
13	6 h 04′ 38″	5 h 34′ 18″	−30′ 20″	−8.32
14	4 h 18′ 32″	4 h 29′ 54″	11′ 22″	4.40
15	4 h 40′ 30″	4 h 47′ 17″	06′ 47″	2.42
16	0 h 39′ 41″	0 h 37′ 09″	−02′ 32″	−6.38
17	6 h 11′ 52″	5 h 28′ 27″	−43′ 25″	−11.68
18	4 h 04′ 03″	4 h 04′ 25″	00′ 22″	0.15
19	1 h 06′ 49″	1 h 04′ 27″	−02′ 22″	−3.54
20	4 h 08′ 26″	3 h 44′ 09″	−24′ 17″	−9.77
Total	82 h 47′ 07″	82 h 44′ 38″	−02′ 29″	−0.05

a stage like 6 has its angles clustered near $\theta = 0$, thus indicating a very flat stage. However, a large spread in angles shows that a stage like 13 is very mountainous.

4. Results and discussion

Table 1 summarizes our 2004 model's main results. While we were once again pleased to come so close to the total of the stage-winning times (just 0.05% discrepancy), we realize that the low discrepancy was aided by cancellations of individual stage discrepancies that are larger than our overall discrepancy. We computed a 1.55% relative error when all of our stage errors were added in quadrature.

We begin our discussion with the four time trial stages (0, 4, 16 and 19). Our predictions for the time trials were much better in 2004 (% differences of 0.24%, 3.84%, −6.38% and −3.54%) than for 2003 (% differences of 20.85%, 14.34%, 13.41% and 18.86%) [1]. Reducing $C_D A$ for stages 0, 4 and 19 and increasing the power for all four time trials greatly improved our predictions. We thus conclude that the effects of drafting, more aerodynamic clothing and bikes, and increased power output are well handled by our model.

There are many factors that we cannot predict ahead of time. One is weather effects. Our prediction for stage 5 was nearly 20 min too fast. We noted while watching the race[14] that a fair amount of rain fell during stage 5 and there was a significant head wind during much of that stage. Many of the lead riders played a 'cat-and-mouse game' before a final sprint near the stage's end. Given the weather, we are not at all surprised by the discrepancy between our fast stage 5 prediction and the actual result.

[14] We watched live coverage of the race on the *Outdoor Life Network*.

Table 2. Numerical results for the 2003 Tour de France. The difference column 4 is the difference between column 3 and column 2. The % difference column 5 is [(column 3 − column 2)/(column 2)] × 100%. The difference column 7 is the difference between column 6 and column 2. The % difference column 8 is [(column 6 − column 2)/(column 2)] ×100%. The actual winning times are taken from Web site given in footnote 3.

Stage	Actual winning time	Predicted time ('03 model)	Difference ('03 model)	% difference ('03 model)	Predicted time ('04 model)	Difference ('04 model)	% difference ('04 model)
0	0 h 07′ 26″	0 h 08′ 59″	01′ 33″	20.85	0 h 07′ 42″	0 h 00′ 16″	3.59
1	3 h 44′ 33″	3 h 43′ 57″	−00′ 36″	−0.27	3 h 55′ 39″	0 h 11′ 06″	4.94
2	5 h 06′ 33″	4 h 35′ 51″	−30′ 42″	−10.01	4 h 50′ 28″	−0 h 16′ 05″	−5.25
3	3 h 27′ 39″	3 h 47′ 02″	19′ 23″	9.33	3 h 58′ 57″	0 h 31′ 18″	15.07
4	1 h 18′ 27″	1 h 29′ 42″	11′ 15″	14.34	1 h 16′ 43″	−0 h 01′ 44″	−2.21
5	4 h 09′ 47″	4 h 29′ 41″	19′ 54″	7.97	4 h 44′ 07″	0 h 34′ 20″	13.75
6	5 h 08′35″	5 h 04′ 35″	−04′ 00″	−1.30	5 h 20′ 51″	0 h 12′ 16″	3.98
7	6 h 06′ 03″	5 h 43′ 22″	−22′ 41″	−6.20	6 h 00′ 58″	−0 h 05′ 05″	−1.39
8	5 h 57′ 30″	6 h 15′ 28″	17′ 58″	5.03	5 h 49′ 13″	−0 h 08′ 17″	−2.32
9	5 h 02′ 00″	4 h 44′ 58″	−17′ 02″	−5.64	4 h 47′ 44″	−0 h 14′ 16″	−4.72
10	5 h 09′ 33″	4 h 41′ 29″	−28′ 04″	−9.07	4 h 55′ 39″	−0 h 13′ 54″	−4.49
11	3 h 29′ 33″	3 h 33′ 12″	03′ 39″	1.74	3 h 45′ 07″	0 h 15′ 34″	7.43
12	0 h 58′ 32″	1 h 06′ 23″	07′ 51″	13.41	0 h 56′ 34″	−0 h 01′ 58″	−3.36
13	5 h 16′ 08″	5 h 26′ 07″	09′ 59″	3.16	5 h 15′ 37″	−0 h 00′ 31″	−0.16
14	5 h 31′ 52″	5 h 23′ 21″	−08′ 31″	−2.57	5 h 24′ 51″	−0 h 07′ 01″	−2.11
15	4 h 29′ 26″	4 h 30′ 38″	01′ 12″	0.45	4 h 27′ 43″	−0 h 01′ 43″	−0.64
16	4 h 59′ 41″	4 h 53′ 21″	−06′ 20″	−2.11	5 h 01′ 35″	0 h 01′ 54″	0.63
17	3 h 54′ 23″	3 h 59′ 45″	05′ 22″	2.29	4 h 12′ 08″	0 h 17′ 45″	7.57
18	4 h 03′ 18″	4 h 35′ 34″	32′ 16″	13.26	4 h 49′ 56″	0 h 46′ 38″	19.17
19	0 h 54′ 05″	1 h 04′ 17″	10′ 12″	18.86	0 h 55′ 04″	0 h 00′ 59″	1.82
20	3 h 38′ 49″	3 h 14′ 55″	−23′ 54″	−10.92	3 h 24′ 58″	−0 h 13′ 51″	−6.33
Total	82 h 33′ 53″	82 h 32′ 37″	−01′ 16″	−0.03	84 h 01′ 34″	1 h 27′ 41″	1.77

Stages 12 and 13 are somewhat enigmatic for us. We missed each of these two stages by roughly the same amount; however, we were too slow in stage 12 and too fast in stage 13. Stage 12 is fairly flat before ending with the first climb in the Pyrenees; stage 13 takes place in the heart of the Pyrenees. It is not clear to us why these two stage predictions missed as they did.

From a percentage standpoint, our prediction for stage 17 was the worst. We have a conjecture for why this occurred. Stage 17 took place the day after stage 16, a gruelling uphill time trial won by Lance Armstrong (see footnote 3). In addition to fatigue that riders must have felt during stage 17, it was a foregone conclusion at the start of stage 17 that Armstrong was on his way to victory. Given that Armstrong was near the front of the pack for a good portion of the stage that he won in the final 25 m (see footnote 3), perhaps there was little incentive for other riders to seriously challenge Armstrong.

We note that stage 20 was our second worst prediction. Unlike in 2003 when he won the Tour de France by just 61 s, Armstrong began the final stage of the 2004 race with an essentially insurmountable 6′ 38″ lead (see footnote 3). Given that Armstrong was sipping champagne en route to the Champs-Elysées and mugging for the camera with a count of six on his fingers (see footnote 14), we understand why our prediction was too fast for a not-so-competitive final stage.

Finally, we show in table 2 our results from the 2003 Tour de France, published in the *American Journal of Physics* [1], along with predictions for the 2003 race using our new 2004 model. Our 2003 model parameters led to a much better total time than that predicted by our 2004 model; however, as we noted in our first paper, [1] the overall sum is as much a fortuitous cancellation of individual stage errors as it is a good choice of model parameters. Even though our 2004 model misses the overall sum of stage-winning times by nearly $1\frac{1}{2}$ h, our prediction is still less than 2% off from the actual total. Moreover, 13 of the individual stage predictions were better using the 2004 model while the other eight were worse. Of the eight 2003 stages that our current model predicted worse than our 2003 model, five of them (stages 1, 6, 10, 15 and 17) still had per cent differences under 10%. Stages 3, 5 and 18 were predicted the most poorly with our 2004 model; all three of our predictions were too slow, just as they were with our 2003 model. It is not clear what made these three stages slow for us.

The fact that our sum of stage-winning times for the 2003 race using our 2004 model came out greater than what our 2003 model predicted should not be surprising. As equation (4) shows, the predominate power used in our 2003 model was 375 W. As equation (5) shows, our 2004 model uses 325 W for most of the terrain covered by the old 375 W power parameter. Thus, the overall predicted time is greater for the 2003 race using our new model compared to our old one.

Probably the most noticeable difference between the two sets of 2003 race predictions deals with the short time trials (stages 0, 4, 12 and 19). Correcting for what we believed was too small a power in our 2003 model for the time trials, equation (6) appears to be a better model of the power output on the time trials. Using our 2004 model, we see that all four of our 2003 time trial predictions missed the actual times by less than 4%, a significant improvement over our 2003 model.

We believe we have demonstrated the utility of our Tour de France model. Using inclined planes to model the terrain and a set of realistic parameters that reflect the physical world and elite cyclist performance, we were able to predict the winning stage times of the 2004 Tour de France to an accuracy under 10% for all stages except one (and that one did not miss 10% by much). Five of our stage predictions came below the 1% level and our overall sum of stage-winning times missed the actual total by just 0.05%. When our 2004 model was applied to the 2003 Tour de France, we missed the overall total by just 1.77%.

Acknowledgments

We wish to thank the paper referee for valuable suggestions for improving our paper. We also thank the referee for suggesting the use of a continuous power function in place of our discrete power functions given in equations (5) and (6). We plan to investigate this interesting suggestion in future work.

References

[1] Hannas B L and Goff J E 2004 Model of the 2003 Tour de France *Am. J. Phys.* **72** 575–9
[2] See, for example, Thornton S T and Marion J B 2004 *Classical Dynamics of Particles and Systems* (California: Brooks/Cole—Thomson Learning) p 59
[3] See, for example, Hecht E 1998 *Physics* 2nd edn (Pacific Grove, CA: Brooks/Cole) p 113
[4] Giordano N J 1997 *Computational Physics* (Englewoods Cliff, NJ: Prentice Hall)
[5] Hennekam W and Govers M 1996 The freewheeling cyclist *Phys. Educ.* **31** 320–8
[6] Padilla S, Mujika I, Angulo F and Goiriena J J 2000 Scientific approach to the 1 h cycling world record: a case study *J. Appl. Physiol.* **89** 1522–7
[7] Reiser R F II, Peterson M L and Broker J P 2001 Anaerobic cycling power output with variations in recumbent body configuration *J. Appl. Biomech.* **17** 204–16
[8] Padilla S *et al* 2001 Exercise intensity and load during mass-start stage races in professional road cycling *Med. Sci. Sports Exerc.* **33** 796–802

[9] Lee H *et al* 2002 Physiological characteristics of successful mountain bikers and professional road cyclists *J. Sports Sci.* **20** 1001–8
[10] Atkinson G, Davison R, Jeukendrup A and Passfield L 2003 Science and cycling: current knowledge and future directions for research *J. Sports Sci.* **21** 767–87
[11] Hug F, Laplaud D, Savin B and Grélot L 2003 Occurrence of electromyographic and ventilatory thresholds in professional road cyclists *Eur. J. Appl. Physiol.* **90** 643–6
[12] Izquierdo M *et al* 2004 Maximal strength and power, muscle mass, endurance and serum hormones in weightlifters and road cyclists *J. Sports Sci.* **22** 465–78
[13] Summers A 2004 Meddling with pedaling *Nat. Hist.* March 42–3