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Parameter space for successful soccer kicks

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Abstract

A computational model of two important types of soccer kicks, the free kick and the corner kick, is developed with the goal of determining the success rate for each type of kick. What is meant by 'success rate' is the probability of getting an unassisted goal via a free kick and the probability of having a corner kick reach an optimum location so that a teammate's chance of scoring a goal is increased. Success rates are determined through the use of four-dimensional parameter space volumes. A one-in-ten success rate is found for the free kick while the corner-kick success rate is found to be one in four.

1. Introduction

The physics of sports has evolved into its own genre within physics research. While US physics journals have published several articles dealing with sports like baseball, American football and basketball, it is quite rare to find an article that deals with soccer[1]. European journals, on the other hand, have in them numerous articles dealing with what could be argued as the world's most popular sport. We believe our work here is best suited for an audience dominated by European scientists where a great deal of quality research on soccer has been recently published [1–7].

Compared to a game like basketball, which typically has a couple of scores per minute of playing time, scoring opportunities in soccer are much rarer. There are, however, opportunities in the course of a soccer game when the chance for a goal rises considerably. We consider in this paper two such opportunities, the free kick and the corner kick.

Free kicks are awarded in various locations for various reasons, such as fouls and touching the ball with a hand. Within about 32.0 m of an opponent's goal, a free kick represents a great scoring opportunity because a player gets 9.14 m of space between the placed ball and the

[1] We use the term 'soccer' in this work since that is the name with which we are most familiar.

defenders[2]. A quintessential example of a perfectly executed free kick, and one that greatly influenced the free kick model we discuss in this work, was made on 26 June 1998 in a World Cup match in Lens, France that pitted David Beckham's England team against Colombia. The right-footed Beckham, famously known for these types of kicks, managed to kick the ball in such a way that it rose over a wall of defenders and curved into the left-most portion of the goal, beyond the goalkeeper's reach.

Corner kicks are awarded when a defender is the last to touch the ball before it crosses the goal line. Taken from one of the two corners near an opponent's goal, corner kick strategy typically involves kicking the ball in such a way that it arrives in front of the goal with the hope that one of the kicker's teammates can then score a goal. Our model for a corner kick thus centres not on having the ball go into the goal, but having the ball enter a certain area in front of the goal.

In our work here, we create a model for a free kick and one for a corner kick. Our free kick model has similarities with the one given in [6]; however, we consider many more kicks and we also include variations in spin rate. We have not found that the corner kick has been studied to the degree that the free kick has. The ultimate goal in this work is to determine reasonable estimates for the success rate of getting an unassisted goal via a free kick and the success rate of having a corner kick reach an optimum location so that a teammate's chance of scoring a goal is increased.

This paper is organized as follows. Section 2 first discusses the physics of the flight of a soccer ball and then moves on to descriptions of the geometries of our model free kick and model corner kick. Section 3 presents the computational results of the parameter space ratios of good kicks to total kicks for both free kicks and corner kicks. Section 4 offers rules of thumb for success rates for both types of kicks.

2. Model description

The physics that describes a soccer ball in flight, i.e. immediately after leaving a player's foot and instantly before coming into contact with any other solid object, is the same for any type of kick, be it a free kick or a corner kick. Before discussing the geometries of the two types of soccer kicks under consideration, we discuss the flight of the ball.

2.1. Flight physics

Several recent papers [1, 2, 4–7] discuss modelling the flight of a soccer ball. Here, we outline the model we used and refer the reader to the references for more details.

While in the air, the soccer ball is subject to a net force given by

$$\vec{F} = \vec{F}_g + \vec{F}_D + \vec{F}_M, \tag{1}$$

where $\vec{F}_g$, $\vec{F}_D$, and $\vec{F}_M$ are the gravitational, drag and Magnus forces, respectively. The gravitational force points straight down and has magnitude mg, where m is the mass of the soccer ball (0.425 kg) and g is the acceleration due to gravity (9.8 m s^{-2}).

The drag force is due to air resistance and points opposite the ball's velocity $\vec{v}$. For the magnitude of $\vec{F}_D$, we use the standard Prandtl expression [8], given by

$$F_D = \tfrac{1}{2} C_D \rho A v^2. \tag{2}$$

[2] We are most familiar with imperial units within the game of soccer; however, we have converted to SI (metric system) since that is the international standard. Thus, the numbers we use in this work originated in the imperial unit system. For example, we have always known the width of a soccer goal to be 8 yards. Therefore, using a width of 7.32 m is simply due to a conversion of units.

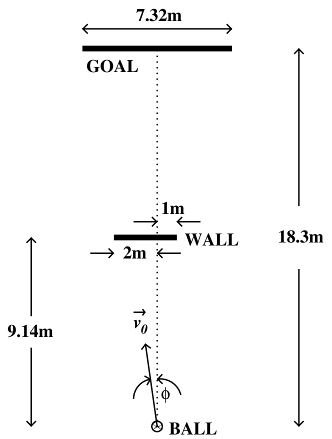

Figure 1. A scale sketch of the top view of the free kick geometry. Two-thirds of the 3 m wide wall lies left of centre. We take the width of the wall to be the width of the ball and its height to be 1.83 m. The goal is 2.44 m tall. The initial velocity of the ball, $\vec{v}_0$, has an azimuthal angle, ϕ, that is positive when measured counterclockwise from the centre line. The polar (or elevation) angle, θ, is measured up from the ground (not shown).

In the above equation, ρ is the air density (1.2 kg m^{-3}), A is the ball's cross-sectional area (0.0388 m^2), and v is the ball's speed. The dimensionless drag coefficient, C_D, is, in general, a function of v. Because no analytic expression exists for that functional dependence, we take C_D to be constant. Based on the experimental work in [6], we take $C_D = 0.275$. This value is quite close to those used by [4] and [9].

The Magnus force is present if the soccer ball is spinning in flight. So-called 'banana kicks' in soccer are due to the Magnus force. Denoting the angular velocity of the ball as $\vec{\omega}$, the Magnus force points along the direction of $\vec{\omega} \times \vec{v}$. To account for the dependence of the magnitude of $\vec{F}_M$ on the ball's angular speed, ω, we use the expression given in [9] for the magnitude of the Magnus force, namely

$$F_M = \tfrac{1}{2} C_M \rho A r \omega v. \tag{3}$$

Here, r is the ball's radius (0.111 m) and we take the dimensionless Magnus coefficient to be $C_M = 1$ [9]. As with C_D, we expect C_M to depend on v; however, since no analytic expression exists, we take C_M to be constant.

To determine the trajectory of the soccer ball, we need to supply our codes with initial launch position and velocity. Then, we numerically solve Newton's second law ($m\vec{a} = \vec{F}$, with $\vec{a}$ the ball's acceleration) using equations (1)–(3). A simple Euler–Cromer algorithm [10] with a time step of 0.001 s works quite well.

2.2. Free-kick geometry

What we describe now is based on our observations of many free kicks in real soccer games. There is no standard placement of players for a free kick; we simply describe what a typical free kick might look like. We imagine a free kick executed with the intent of scoring a goal without help from a teammate. The initial position of the ball is 18.3 m from the centre of the goal. The rules dictate that a wall of defenders can be placed 9.14 m from the location of the free kick so as to provide an obstacle to scoring. See figure 1 for a top view of the geometry. We note that our wall geometry is simpler than that used in [6].

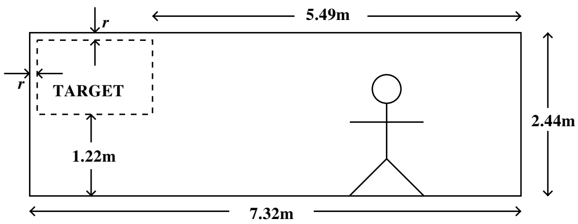

Figure 2. A scale sketch of a regulation soccer goal with our model target. The goal is 7.32 m wide and 2.44 m tall. The target occupies the upper-left quadrant of the left half of the goal (as seen by the kicker), though it must be offset by the ball's radius, *r*, from the top and left edges. For perspective, a 1.83 m goalkeeper is shown guarding the right portion of the goal.

We go with the majority of soccer players and use a right-footed kicker. A right-footed kicker typically tries to put counterclockwise spin on the ball (as viewed from above). The Magnus force then leads to a curve to the player's left. Our model kicker thus tries to make the ball curve from right to left and enter the upper-left portion of the goal (as seen by the kicker).

To defend the kick, we place the wall left of centre and allow the goalkeeper to defend the right half of the goal. We choose a width of 3 m for the wall. Our target for a free kick goal is a rectangle in the upper-left portion of the goal. We imagine the goalkeeper being able to defend the right half of the goal and half of the left half of the goal. The wall could defend the lower left quarter of the goal, as well as the goalkeeper (if the player kicked the ball at such a large launch angle that the time of flight was long enough to allow the goalkeeper to get to that part of the goal). See figure 2 for an illustration of the goal and target.

2.3. Corner-kick geometry

For the corner kick, our model player is not trying to kick the ball directly into the goal. Instead, we employ a popular soccer strategy whereby the ball is kicked with side-spin so as to make the ball move away from the plane of the goal with the intention of being knocked into the goal by a teammate's head, or possibly a so-called 'bicycle' kick. Thus, our model kicker is not aiming for a two-dimensional target in the plane of the goal, but rather an imagined three-dimensional volume in front of the goal. Our target is in the vicinity of a player's head and we set its vertical dimension to be between 1.68 m and 2.44 m. We further assume that our target is as long as the goal (7.32 m) since a teammate can head the ball into any part of the goal; however, we offset the target 0.914 m away from the kicker to better utilize the angle of the incoming kick. Finally, our target is 1.83 m wide to account for the typical spacing of players waiting for the corner kick to arrive. As with the free-kick target, our choice of target for the corner kick is somewhat arbitrary; however, we feel it is well motivated by the numerous corner kicks in actual soccer games we have viewed. We also note that there is some variance in the width of a soccer field [11]. Our model field is 68.6 m wide.

Figure 3 shows a top view of our corner-kick geometry. We again imagine a right-footed player kicking the ball such that it spins counterclockwise (as viewed from above) so as to make the ball move away from the plane of the goal. The ball then possesses a component of velocity toward a teammate trying to head the ball in for a goal.

3. Results and discussions

The trajectory of a given kick is determined by four free parameters: v_0, the ball's initial speed, ω, the ball's rotational speed (assumed constant throughout the ball's flight), θ, the

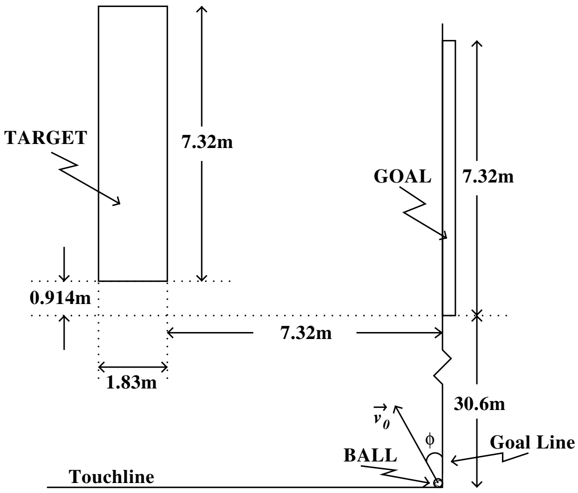

Figure 3. A not-to-scale top view sketch of our model target for a corner kick. The target is a parallelepiped with dimensions 7.32 m × 1.83 m × 0.762 m and sits 1.68 m above the ground. The goal is 2.44 m tall and 1.22 m deep. The ball is kicked 0.457 m in and 0.457 m up from the lower right corner with an initial velocity, $\vec{v}_0$. The azimuthal angle, ϕ, is positive when measured counterclockwise from the goal line. The polar (or elevation) angle, θ, is measured up from the ground (not shown).

initial launch angle (measured from the ground) and ϕ, the azimuthal angle (see figures 1 and 3). We take the spin axis of the ball, i.e. the direction of $\vec{\omega}$, to point straight up[3] for all kicks, meaning the ball is spinning counterclockwise as viewed from above.

There are certainly physical limits to the values we can choose for v_0, ω, θ, and ϕ. For example, a player generates spin on the ball when the ball is kicked along a line that does not pass through the ball's centre. More spin can be generated if the foot strikes the ball closer to the ball's edge. However, as the foot moves away from a line passing through the ball's centre, the maximum launch speed goes down [3].

For each of the two types of kicks under consideration, we consider a reasonable range for the four free parameters. We imagine a professional soccer player approaching a ball with an idea in mind as to how the kick is to be executed. Despite being a professional, there will always be some error in how the ball is kicked. Missing a spot on the ball that a player hopes to kick by just one or two centimetres can significantly alter the ball's launch velocity and spin rate [3].

Given a reasonable set of ranges for v_0, ω, θ, and ϕ, we then construct a four-dimensional parameter space volume in which each point contained within the volume corresponds to a specific kick. The total number of discrete points in our parameter space represents the total number of kicks. We assume that our parameter ranges are small enough that we can use equal *a priori* probabilities. In other words, each kick is equally likely. We then run our codes for every kick and count the number of 'good' kicks. In the case of a free kick, that corresponds to the ball hitting the target in figure 2 while avoiding the wall shown in figure 1. For a corner kick, a good kick is one that hits the three-dimensional target shown in figure 3.

Once all kicks have been evaluated and the good kicks counted, we determine the fraction of the four-dimensional parameter space volume occupied by good kicks. That fraction is

[3] Our codes allow for the rotation axis to point in any direction we like. To simplify the discussion in this work, we take the rotation axis to point straight up.

simply the number of good kicks divided by the total number of kicks. The assumption of equal *a priori* probabilities makes for such a simple calculation of the fraction of good kicks. It is that fraction that we use as our estimate for the level of difficulty of executing a successful kick.

To be absolutely clear, the fact that even a professional soccer player is not capable of hitting a ball at exactly the desired place on the ball each and every time the player kicks the ball means that there is uncertainty in the parameters associated with a given kick. Thus, the error in this work is associated with a player's inability to achieve perfection with every kick. It is that inability that leads to ranges for our four parameters; i.e., the starting point for our computational work is with the error associated with kicking a soccer ball. Once the parameter ranges are determined, we simply run our codes using our model geometries. We have some integer number of total kicks and some smaller integer number of kicks that hit our model targets.

We now examine the parameter space for successful free kicks. After that, we will examine the corner kick.

3.1. Free-kick parameter space

Based on previously measured ball speeds [4, 6] and spin rates [3], and based on the playing experience of one of us (BGC), we have determined the following ranges as reasonable for a free kick[4]: 24.6 m s^{-1} $\leqslant v_0 \leqslant$ 29.1 m s^{-1}, 6 rev s^{-1} $\leqslant \omega \leqslant$ 12 rev s^{-1}, $13^\circ \leqslant \theta \leqslant 17^\circ$, and $1^\circ \leqslant \phi \leqslant 5^\circ$. Smaller values of v_0 and larger values of θ could lead to kicks that hit our target; however, the times of flight of those kicks exceed one second. Flight times longer than about one second most likely give the goalkeeper the opportunity to make a save. Note that 'revolutions' and 'degrees' must be converted to radians. Our discrete four-dimensional mesh uses the following step sizes (see footnote 4): $\Delta v_0 = 0.0447$ m s^{-1}, $\Delta\omega = 0.1$ rev s^{-1}, $\Delta\theta = 0.1^\circ$, and $\Delta\phi = 0.1^\circ$. Thus, given the ranges stated above, there are a total of $101 \times 61 \times 41 \times 41 = 10\,356\,641$ points (or kicks) in the four-dimensional parameter space volume.

We found that 9.63% of all free kicks missed the wall of defenders and hit the target. In other words, 9.63% of the four-dimensional parameter space is occupied by successful kicks. Creating a finer mesh in our four-dimensional space does not change that percentage at the level of three-digit accuracy. Since a four-dimensional volume cannot be displayed, we instead show in figure 4 a three-dimensional slice of the four-dimensional parameter space volume. In that plot, we fixed the one parameter, ϕ, that we thought would be the easiest for a player to control.

Figure 5 shows one of the successful free kicks contained in the parameter subspace volume shown in figure 4. One can easily see the effect of the Magnus force as the soccer ball curves from right to left and enters our target. To illustrate the importance of being able to impart spin on a soccer ball during a free kick, we also show in figure 5 a kick with no spin. While that kick clears the wall of defenders, it can be successfully defended by the goalkeeper because it enters the plane of the goal near the centre. The curved kick not only reaches a part of the goal that a goalkeeper is not likely to reach, its trajectory can fool the goalkeeper into initially thinking the ball will follow the no-spin trajectory. The same idea applies in a sport like baseball where pitchers throw curve balls to fool batters.

[4] So that our speeds do not seem so strange, note that what we had in mind initially was 55 miles/hour $\leqslant v_0 \leqslant$ 65 miles/hour with a step size of $\Delta v_0 = 0.1$ miles/hour. We have simply converted miles/hour to m s^{-1}.

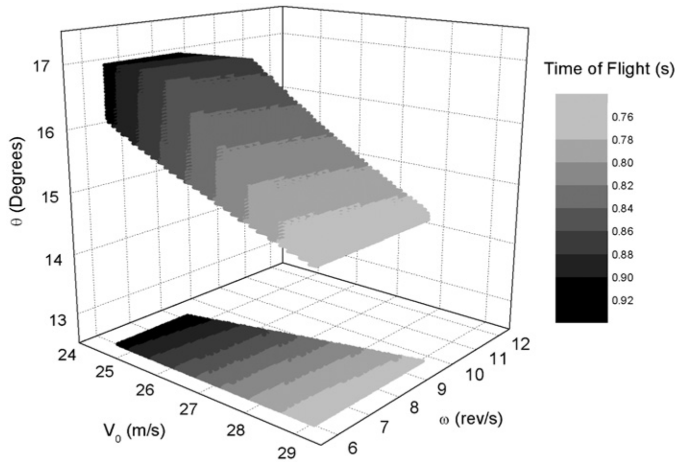

Figure 4. The parameter subspace of successful free kicks. The azimuthal angle is $\phi = 3^\circ$ (see figure 1). The grey scale on the right represents time of flight values.

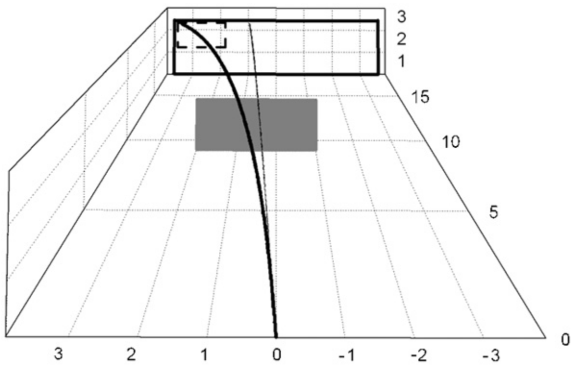

Figure 5. A trajectory of a free kick. The heavy line shows the trajectory for one of the successful kicks contained in the subspace plot in figure 4. For the heavy line trajectory, $v_0 = 28.6$ m s^{-1}, $\omega = 9$ rev s^{-1}, $\theta = 15^\circ$ and $\phi = 3^\circ$. The light line shows a free kick with the same v_0, θ, and ϕ as the heavy line trajectory; however, $\omega = 0$. While that no-spin kick clears the wall of defenders, it misses the target; i.e., the goalkeeper is able to keep it from entering the goal. All distance values are in metres.

3.2. Corner-kick parameter space

The corner kick must cover a greater distance than the free kick. Since the initial speed range we chose for the free kick probably represents a reasonable range for the corner kick, we decided to achieve the greater distance with larger launch angles. We also assumed that, in an attempt to maintain as much control as possible, a player executing a corner kick is not likely to use as large a rotational speed as in the free kick. The parameter ranges we settled on are 24.6 m s$^{-1} \leqslant v_0 \leqslant 29.1$ m s^{-1}, 3 rev s$^{-1} \leqslant \omega \leqslant 9$ rev s^{-1}, $24^\circ \leqslant \theta \leqslant 28^\circ$, and $-2^\circ \leqslant \phi \leqslant 2^\circ$. As with the parameter ranges for the free kick, personal playing experience by one of us (BGC) assisted us in making our choices. In creating the four-dimensional mesh, we used the exact same step sizes that we used for the free kick. Thus, since the corner

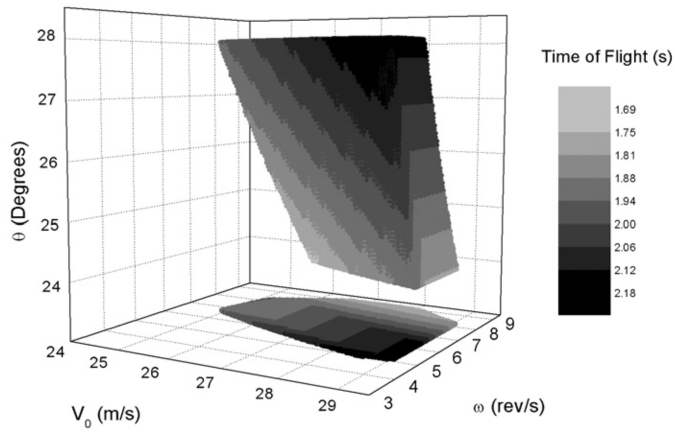

Figure 6. The parameter subspace of successful corner kicks. The azimuthal angle is $\phi = -1^\circ$ (see figure 3). The grey scale on the right represents time of flight values.

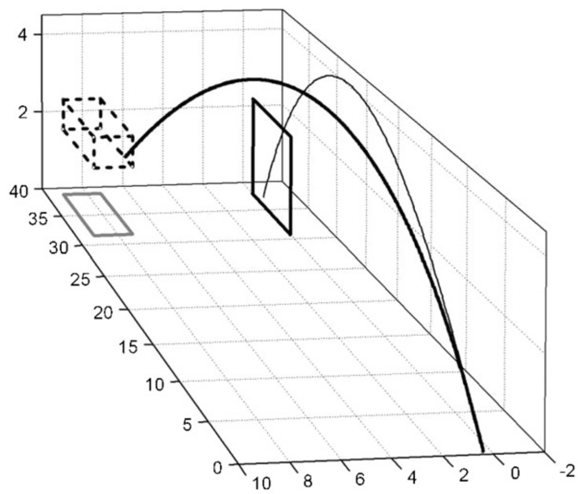

Figure 7. A trajectory of a corner kick. The target, represented by the dashed parallelepiped, is shown with its projection on the ground. The heavy line shows the trajectory for one of the successful kicks contained in the subspace plot in figure 6. For the heavy line trajectory, $v_0 = 27.3$ m s^{-1}, $\omega = 7$ rev s^{-1}, $\theta = 24^\circ$ and $\phi = -1^\circ$. The light line shows a corner kick with the same v_0, θ, and ϕ as the heavy line trajectory; however, $\omega = 0$. That no-spin kick actually hits behind the plane of the goal and is thus out of bounds. All distance values are in metres.

kick ranges are all the same size as those for the free kick, we once again have a total of 10 356 641 points (or kicks) in the four-dimensional parameter space volume.

We found that 24.5% of all corner kicks hit the three-dimensional target. Since the corner-kick target is wider than the target for the free kick, and the corner-kick target is three dimensional, we expect a higher percentage of successful kicks for the corner kick. Note that a successful corner kick for us is one that enters the three-dimensional target shown in figure 3. We are not claiming anything about whether or not a given corner kick ends up as a goal. That is a separate problem. Our interest in this work lies in determining the chance of success for the player who kicks the ball.

As with the free kick, we show in figure 6 a three-dimensional slice of the four-dimensional parameter space volume. Because the corner kick is longer than the free kick, the corresponding flight times are larger. Figure 7 shows one of the successful corner kicks contained in the parameter subspace volume shown in figure 6. A corner kick without spin is also shown. Note that the good corner kick has a component of its velocity pointing away from the plane of the goal, meaning, as previously discussed, a teammate could then head the ball back toward the goal.

4. Conclusions

We have used ranges for four parameters that we feel reasonably reflect typical error ranges for a professional soccer player executing either a free kick or a corner kick. From those ranges, we were able to determine the chance of success for both types of kicks by determining the ratio of good kick parameter space volume to total kick parameter space volume. Given the complexities of the game of soccer and the relative crudeness of our model, we offer here merely estimates, or rules of thumb, of the success rates for free kicks and corner kicks. Only about one in ten free kicks of the type we described in this work results in unassisted goals. The corner kick success rate is higher, roughly one in four.

The aforementioned rules of thumb represent the level of accuracy we can achieve with our predictions. We only stated three-digit accuracy in the previous section because we were simply dividing two numbers from our code. We are certainly not trying to claim that level of accuracy with the kind of rules of thumb we have established. While we chose ranges of parameters we felt were physically reasonable based on previously published work [3] and our own observations, we realize that other researchers might choose different ranges. A different choice could easily alter our rules of thumb. For example, it can be seen in figure 4 that there are no successful kicks for spin rates greater than roughly 9 rev s^{-1}. Had we taken ω all the way to 18 rev s^{-1}, for example, we would have had no additional successful kicks. However, we would have increased the total four-dimensional parameter space volume by a factor of two, thus changing our rule of thumb for good free kicks to be one in twenty instead of one in ten. We simply did not wish to use a spin rate larger than 12 rev s^{-1} because we did not believe that a larger range would be physically reasonable for a professional soccer player.

Soccer is a sport that does not keep vast arrays of statistics, in contrast to a sport such as baseball. Despite contacting several professional leagues, we have been unable to find good statistical data for unassisted free kick goals[5]. We doubt statistics are kept regarding any type of defined success rate for corner kicks. We can offer, however, that based on the numerous soccer games we have viewed, the two rules of thumb that we found appear to be quite reasonable.

In future work, we will pursue the issue of how best to weight each parameter. We believe the parameter ranges we have chosen to be small enough that the assumption of equal *a priori* probabilities is good enough to yield reasonable rules of thumb. However, future work is needed to determine appropriate weighting factors if the parameter ranges are to be increased beyond what we have used in this work.

[5] If any reader knows of the existence of such statistics, we would greatly appreciate being informed of them.

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[11] For diagrams of soccer fields using both imperial and metric units, see, for example, www.fifa.com