Model of the 2003 Tour de France

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We modeled the 2003 Tour de France bicycle race using stage profile data for which elevations at various points in each stage are known. Each stage is modeled as a series of inclined planes. We accounted for the forces on a bicycle-rider combination such as aerodynamic drag and rolling resistance and calculated the winning stage times for an assumed set of bicycle and rider parameters. The calculated total race time differed from the sum of all actual winning stage times by only 0.03%.
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I. INTRODUCTION

Over the years, several excellent articles[1–11] have applied physics to the study of bicycles. The work described in those articles is particularly suited for undergraduate projects. We wish to contribute to this body of work with our study which grew out of an undergraduate (BLH) project in a computational physics course. We were motivated by some ideas for modeling a bicycle in motion given in Ref. 12, and we modeled the 2003 Tour de France.

At the time of this writing, Lance Armstrong had just won his record-tying fifth straight Tour de France by the narrow margin of 61 seconds. We modeled each stage of the race with the goal of predicting the winning time. Accounting for aerodynamic drag and rolling resistance, we determined the various stage times by using a simple two-dimensional model of the course terrain. Our goal was to select model parameters that were motivated by the literature and the course terrain and to avoid having to adjust the various parameters at the race's conclusion.

This paper is organized as follows. Section II discusses the models used to analyze the finishing times of the 2003 Tour de France. Section III presents the computational results of the terrain model, and Sec. IV discusses the results of the bicycle model.

II. MODEL DESCRIPTION

The motivation for our model came after examining dynamic profiles of the 2003 Tour de France.[13] Figure 1 shows a typical profile, that of stage 15. The distances shown in kilometers refer to the actual distances the cyclists must travel, despite the fact that these distances are shown on the horizontal.[14] These distances are given to the nearest 0.5 km. The elevations above sea level at various points along the way are given in meters. Although the actual geometry of a Tour de France stage is three-dimensional with winding turns, we modeled the stage by using only the two-dimensional slice given in Fig. 1. A knowledge of the elevations at the points specified on Fig. 1 means that we can make only a very rough model of a stage's true profile, and we modeled a stage by using a linear approximation of the terrain between the known elevations. Thus, our model's stage profile looks like a series of inclined planes (see Fig. 2).

For a given inclined plane, we modeled the bicycle-rider combination of total mass m as one object subjected to a number of forces. The free-body diagram and our coordinate system is shown in Fig. 3. The weight of the bicycle-rider combination is $m\vec{g}$, where $\vec{g}$ is the acceleration due to gravity. The normal force, $\vec{F}_N$, is the component of the ground's force on the bicycle-rider that is perpendicular to the ground.

We also included two frictional forces. The first is due to aerodynamic drag, which we assumed to be proportional to the square of the cyclist's speed; we neglected the linear drag. Specifically,

$$F_D = \tfrac{1}{2} C_D \rho A v^2, \tag{1}$$

is the drag force,[15,16] where C_D is the dimensionless drag coefficient, ρ is the air density, A is the frontal cross-sectional area of the bicycle-rider combination, and v is the speed. In general, C_D depends on v because it is merely a measure of how the drag force varies from the dynamical pressure multiplied by the cross-sectional area, but we assume that C_D is a constant, although it will vary during the course of a race. C_D has typical values in the range of 0.8–0.9 for a racing cyclist.[17] The cross-sectional area, A, is another parameter that changes during a race. We assumed it to be constant for a given racing position, though we changed its value depending on whether the cyclist was going uphill or downhill. (See Sec. III for the values we used.)

We also modeled the frictional force due to rolling resistance.[18] We considered only a constant rolling resistance force and neglected any terms proportional to v or higher powers of v. The form we used is

$$F_r = \mu_r F_N, \tag{2}$$

where μ_r is the coefficient of rolling friction and F_N is the magnitude of the normal force. This force will be important at low speeds, like those encountered during steep uphill ascents; however, it will be small compared to F_D at high speeds, like those reached when cycling down a steep hill. Typical values of μ_r are around 0.003 for a racing bike.[19]

Finally we considered the forward parallel force, $\vec{F}_b$, on the bicycle-rider due to the reaction of the bike's tires pushing back on the road. We modeled this force by estimating the amount of power, P_b, that the rider puts into the bike so that F_b is

$$F_b = P_b / v. \tag{3}$$

Because Eq. (3) diverges for small v (due to the incorrect assumption that the power remains constant), we corrected Eq. (3) by replacing v with 6 m/s if $v < 6$ m/s. This correc-

Fig. 1. Dynamic profile of stage 15 of the 2003 Tour de France (Ref. 13).

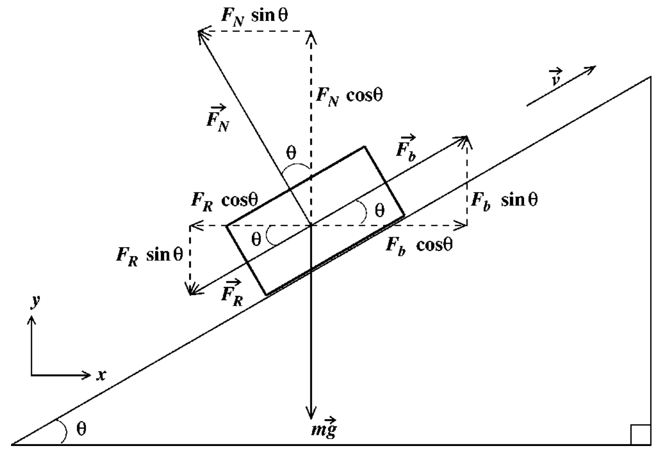

Fig. 3. Free-body diagram of the bicycle-rider combination (represented by the rectangle). Note the coordinate system we used in the lower left. Also note that $\vec{F}_R$ is the sum of the aerodynamic drag force and the rolling friction force.

tion was suggested in Ref. 12. We note that Giordano[12] used 7 m/s, but we estimated that a Tour de France cyclist would be able to maintain a slightly larger force.

In Fig. 3 we represent all of the resistive forces as $\vec{F}_R$. We have,

$$\vec{F}_R = \vec{F}_D + \vec{F}_r . \tag{4}$$

Unlike the usual introductory physics approach, we chose not to rotate the coordinate system with x along the plane and y normal to the plane so that we needed only to change θ when moving from one inclined-plane segment of a stage to the next. The magnitude of the normal force is $F_N = mg\cos\theta$. Newton's second law for the cyclist's acceleration gives

$$\dot{v}_x = \ddot{x} = \left(\frac{F_b}{m} - \frac{F_R}{m} - g\sin\theta\right)\cos\theta, \tag{5a}$$

$$\dot{v}_y = \ddot{y} = \left(\frac{F_b}{m} - \frac{F_R}{m} - g\sin\theta\right)\sin\theta. \tag{5b}$$

A dot represents differentiation with respect to time and v_x and v_y are the x and y components, respectively, of the cyclist's velocity. We need now only insert Eqs. (1) and (2) into Eq. (4) and then substitute this result with Eq. (3) into Eq. (5).

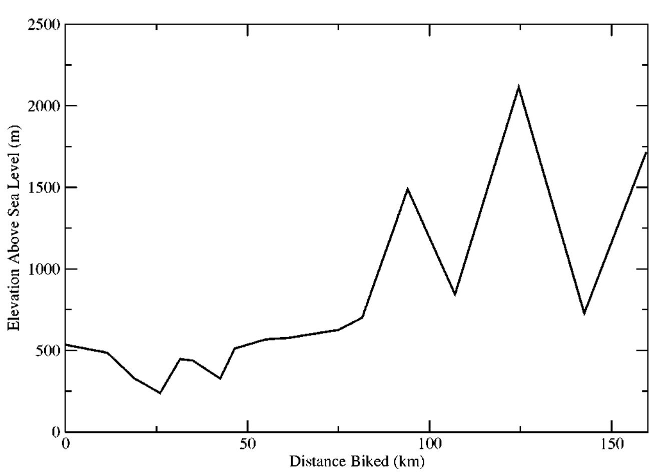

Fig. 2. Our model of stage 15 of the 2003 Tour de France.

III. RESULTS

To obtain a predicted time for each stage of the race, we must numerically solve Eq. (5) for a given stage profile. Thus, the angle θ above (or below) the elevation for each triangular segment is needed. Starting at zero velocity, the cyclist enters the first segment. What gets the cyclist initially moving is obviously the mechanical power put into the system by the rider. Adding up the time for each inclined part of the stage profile will then give the total time for the stage.

The input parameters have a range of values that fit many cyclists at the Tour de France. We used as many published values as we could find and averaged the range when the range of a particular parameter was sufficiently large. One such parameter is the mass[13] and we estimated a mass of 69 kg for the rider and 8 kg for the bicycle, thus giving $m = 77$ kg for the bicycle-rider combination. The values of C_D and A vary from rider to rider; we estimated two values for the product $C_D A$, depending on whether the cyclist is riding downhill in a more crouched position or riding uphill in a more upright stance. From the work of Refs. 17, 20, and 21, we chose $C_D A = 0.25\ \mathrm{m}^2$ for downhill and $C_D A = 0.35\ \mathrm{m}^2$ for uphill. We took the air density to be $\rho = 1.2\ \mathrm{kg/m}^3$ and the coefficient of rolling friction to be $\mu_r = 0.003$ (see, for example, the work of Ref. 19).

Choosing the biker's power output is another parameter that can be estimated. We assumed that the cyclist outputs more energy pedaling uphill than riding downhill. In fact, video footage of this year's Tour de France showed many of the riders coasting on stretches of some of the steep descents. Overcoming the gravitational force on the uphill segments requires most of the rider's energy output. To account for this difference in power output, we examined all of the angles in all of the stages and looked for regions of separation. The steepest climb was around $\theta \simeq 0.097$ rad found in a 2 km stretch near the end of stage 13 while steep descents appeared below $\theta = -0.055$ rad. Given these angular divisions, we arrived at the following estimate for the power outputs

$$P_b = \begin{cases} 200\ \mathrm{W} & (\theta \leqslant -0.055) \\ 375\ \mathrm{W} & (-0.055 \leqslant \theta < 0.09), \\ 500\ \mathrm{W} & (0.09 \leqslant \theta) \end{cases} \tag{6}$$

Table I. Numerical results. The difference column is the difference between columns 3 and 2. The % difference is $[(\text{column }3-\text{column }2)/(\text{column }2)]\times 100\%$. The actual winning times are taken from Ref. 13.

Stage	Actual winning time	Predicted time	Difference	% Difference
0	0 h 07′ 26″	0 h 08′ 59″	01′ 33″	20.85
1	3 h 44′ 33″	3 h 43′ 57″	−00′ 36″	−0.27
2	5 h 06′ 33″	4 h 35′ 51″	−30′ 42″	−10.01
3	3 h 27′ 39″	3 h 47′ 02″	19′ 23″	9.33
4	1 h 18′ 27″	1 h 29′ 42″	11′ 15″	14.34
5	4 h 09′ 47″	4 h 29′ 41″	19′ 54″	7.97
6	5 h 08′ 35″	5 h 04′ 35″	−04′ 00″	−1.30
7	6 h 06′ 03″	5 h 43′ 22″	−22′ 41″	−6.20
8	5 h 57′ 30″	6 h 15′ 28″	17′ 58″	5.03
9	5 h 02′ 00″	4 h 44′ 58″	−17′ 02″	−5.64
10	5 h 09′ 33″	4 h 41′ 29″	−28′ 04″	−9.07
11	3 h 29′ 33″	3 h 33′ 12″	03′ 39″	1.74
12	0 h 58′ 32″	1 h 06′ 23″	07′ 51″	13.41
13	5 h 16′ 08″	5 h 26′ 07″	09′ 59″	3.16
14	5 h 31′ 52″	5 h 23′ 21″	−08′ 31″	−2.57
15	4 h 29′ 26″	4 h 30′ 38″	01′ 12″	0.45
16	4 h 59′ 41″	4 h 53′ 21″	−06′ 20″	−2.11
17	3 h 54′ 23″	3 h 59′ 45″	05′ 22″	2.29
18	4 h 03′ 18″	4 h 35′ 34″	32′ 16″	13.26
19	0 h 54′ 05″	1 h 04′ 17″	10′ 12″	18.86
20	3 h 38′ 49″	3 h 14′ 55″	−23′ 54″	−10.92
Total	82 h 33′ 53″	82 h 32′ 37″	−01′ 16″	−0.03

where the last entry is used only on the aforementioned 2 km stretch of stage 13. We chose that number based on finding the minimum power needed to reach the top of the inclined segment.[22] The other power numbers are consistent with those found in the literature.[9,21,23,24] We originally used 400 W for the intermediate angular range, but changed it to 375 W about midway through the actual race because it seemed as if we were overestimating the power input on the more flat terrain. This was our only parameter change during the course of the actual race. We note that there are a total of 401 angles given on the Tour de France web site[13] for the entire 3427.5 km race. There were 15 angles ($\simeq 3.74\%$ of total) for which $\theta \leqslant -0.055$ rad. The distance which these 15 angles covered was 124.5 km ($\simeq 3.63\%$ of the total distance). Most of the race (3301 km) had 385 angles in the middle range.

Our numerical technique employed the Euler method (see, for example, Ref. 25); nothing more sophisticated is necessary. We found an unchanging stage time when using a time step size of $\Delta t = 0.5$ s. Table I displays our main results.

We stress that the times we computed for Table I were found by starting the biker off at zero velocity and then running our code to the end of a stage. There will obviously be changes in the cyclist's speed as the angle changes and the biker moves from one inclined plane to another. Figure 4 shows a plot of the biker's speed versus distance traveled for stage 15. Note that while the biker's speed changes continuously from one inclined plane to the next, the biker's speed reaches a constant rather quickly for a given inclined plane. This constant is just the terminal speed. For stage 15, we found a maximum speed on a downhill of about 20.9 m/s (46.7 mph). Thus, a faster method of getting an estimate of the time to complete a stage is to determine the terminal speed for a given inclined plane and then assume that that speed is the biker's speed for the entire inclined plane. That is, just solve for the biker's speed in Eq. (5) for $\ddot{x} = \ddot{y} = 0$. This approximation takes about 10–20 s off the more accurately determined stage times, unless there were particularly steep mountains. In that case, the biker approaches a steep mountain at a speed greater than the terminal speed and hence the average speed for that inclined plane is greater than the terminal speed. Stages 7–9 and 13–16 gave slightly longer times (<1 min) when using just the terminal speed for each inclined plane in comparison with the full calculation.

IV. DISCUSSION

As stressed in Sec. I, we intended to model the 2003 Tour de France with a given set of parameters and to see how closely we could predict each stage's winning time. We did not want to continually adjust all the model parameters after each stage was completed just to get a few minutes closer to

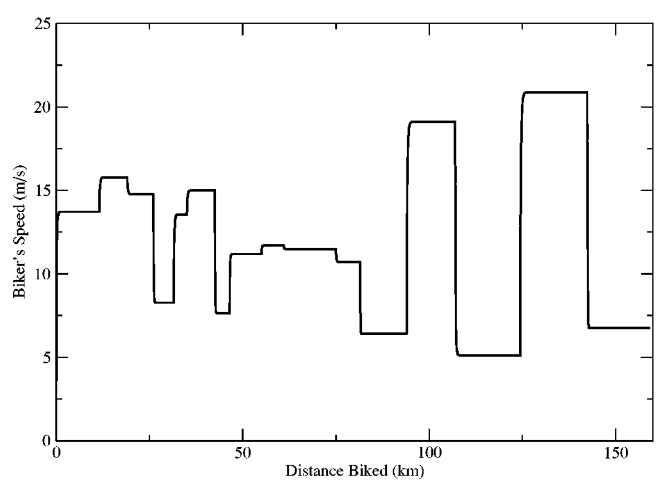

Fig. 4. Our model's prediction of the rider's speed versus distance biked for stage 15 of the 2003 Tour de France.

the winning time. Of the 21 stages we modeled, 9 of our predictions came in faster than the actual winning times and 12 came in slower. With an overall discrepancy of 0.03%, we believe our model was quite successful. The fact that our discrepancy is so low is probably due as much to a fortuitous cancellation of individual stage discrepancies, for which the discrepancies were never as low as 0.03%, as to a judicious choice of model parameters at the beginning of the race. In fact, adding each of our twenty-one stage errors in quadrature gives an overall relative error of 1.54%. Note that the total actual winning time given in Table I is the sum of all the stage-winning times. It is not Lance Armstrong's overall winning time, which was 83 h 41′12″.[13] We were 1 h 8′35″ faster than Armstrong's time, or about 1.37% off. From Ref. 13 we found a mass for Lance Armstrong of 75 kg, which is 8 kg more than what we used.

Our worst results were for the short time trials. The times we computed for stages 0, 4, 12, and 19 were all slow and in the 13%–21% error range. We used a power of 375 W exclusively for these flat stages. We believe that our power was too small; that is, the actual cyclists were likely generating larger power outputs over the short courses. Large power outputs are not likely sustained for the longer stages. A more sophisticated model would use a power larger than 375 W for the time trials.

Interpreting the effects of altering the model's other parameters is straightforward. For example, increasing C_D, A, ρ, and μ_r would increase the resistive drag on the cyclist. Hence, an increase of any of those parameters would result in longer stage times.

Changing the mass of the bicycle-rider combination leads to more subtle effects. An increased mass means that more power must be exerted by the rider to keep climbing speeds the same. However, an increased mass typically means a rider can exert more power, which could improve downhill times. See Ref. 20 for a study on how body size influences uphill and downhill biking.

Of all the parameters (P_b, m, C_D, A, ρ, and μ_r), we found that the stage times were most sensitive to changes in the rider's power output. For example, we found that increasing the power output by 5% gave a time for stage 15 that was about 14 min faster (5% faster), while decreasing the power by 5% slowed the estimated time by about 35 min (13% slower). Clearly, the biker suffers on steep climbs if we reduce the power. However, we note that our model is not very sensitive to our choice of 200 W for the steep downhills. Obviously, the biker's speed is largest on those downhills as is the drag force. Very little is gained by adding power on steep downhills because the drag force scales with the square of the speed.

We should also mention that our study did not model an individual cyclist in the 2003 Tour de France. If one would like to use our model for a particular cyclist, we would recommend doing a power analysis of that rider for various types of hills and straightaways so as to replace Eq. (6) with a more individualized power breakdown. Also, photographs could be taken to deduce a biker's cross-sectional area in various biking positions. The coefficient of rolling friction, μ_r, for a particular tire could be measured in a basic physics lab.[9] One could even model the effect of rolling resistance by including another term that is proportional to the bike's speed.[17] C_D is much more difficult to estimate and could only be accurately known through experimental tests that examine the cyclist's resistive forces. Of course, surveying the land for a particular race course and making air density measurements would improve the estimates of the various θs and ρ, respectively.

All of these suggestions for improving the estimates of the model's parameters would make our model much more sophisticated. There are other levels of sophistication, however, that could be incorporated into our model that would still leave it relatively simple. For example, the effect of wind could be included by simply changing v^2 in Eq. (1) to $(v-v_w)^2$, where the wind velocity, v_w, is taken to be parallel to the ground with $v_w>0$ for a tailwind and $v_w<0$ for a headwind. For a race as long as the Tour de France, an accurate inclusion of wind would be quite challenging, as would obtaining more accurate values for A, C_D, and ρ.

Lastly, there are aspects to actual racing that are quite difficult to include in our model. For example, we have not included drafting. Drafting occurs when one racer rides close behind another racer so as to reduce the amount of aerodynamic drag. Energy savings for a rider in a drafting position can be as high as almost 40%.[26] Although drafting is not so important on steep uphill climbs, we expect our model to predict rather different times for steep downhill descents if we were to include drafting. One could model drafting by using, for example, an effective area that would be smaller than the actual cross-sectional area of the bicycle-rider combination. Or, one could reduce the drag coefficient to a value appropriate for drafting. We also would find it difficult to model the various food and restroom breaks, both of which can take place while the biker is in motion.

We could spend more time adjusting the parameters to more closely approach the winning stage times; however, there are too many approximations to believe that the adjusted parameters would reflect reality any better than the ones we have used to produce Table I. With the amount of readily available online information,[13] there are countless possibilities available to instructors for student projects. Whether the course is computational physics, the physics of sports, classical mechanics, or even introductory physics, a model such as the one we used in this work is well suited for an undergraduate physics student. Any sporting event that has profile data available, including future Tour de France races, can be studied using a model similar to the one we used here. Long-distance automotive races and dog-sled races are examples of less traditional sports that could be studied with our model; all one needs is a good estimate of the various parameters and profile data that could be used in a series of inclined-plane motions.

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[1]J. Fajans, "Steering in bicycles and motorcycles," Am. J. Phys. **68**, 654–659 (2000).

[2]Ricardo E. de Souza and Giovani L. Vasconcelos, "Visual appearance of a rolling bicycle wheel," Am. J. Phys. **64**, 896–897 (1996).

[3]Cliff Frohlich, "Effect of wind and altitude on record performances in foot races, pole vault, and long jump," Am. J. Phys. **53**, 726–730 (1985), Sec. V.

[4]J. Lowell and H. D. McKell, "The stability of bicycles," Am. J. Phys. **50**, 1106–1112 (1982).

[5]Daniel Kirshner, "Some nonexplanations of bicycle stability," Am. J. Phys. **48**, 36–38 (1980).

[6]J. Liesegang and A. R. Lee, "Dynamics of a bicycle: Nongyroscopic aspects," Am. J. Phys. **46**, 130–132 (1978).

[7]William M. Wehrbein, "Collecting and recording bicycle speed data by CBL," Phys. Teach. **41**, 243–244 (2003).

[8]Manfred Euler, Gert Braune, Soenke Schaal, and Dean Zollman, "Collect-

ing kinematics data over long time intervals,'' Phys. Teach. **38**, 434–436 (2000).
[9]Chris Waltham and Brian Copeland, ''Power requirements for rollerblading and bicycling,'' Phys. Teach. **37**, 379–382 (1999).
[10]Nathan H. Rich, ''Bicycle computers and kinematics,'' Phys. Teach. **27**, 627–628 (1989).
[11]Robert G. Hunt, ''Bicycles in the physics lab,'' Phys. Teach. **27**, 160–165 (1989).
[12]Nicholas J. Giordano, *Computational Physics* (Prentice Hall, New Jersey, 1997).
[13]The Tour de France Web site is ⟨www.letour.fr⟩. (We used the English version.)
[14]Private communication with Jeff Jones, editor of ⟨cyclingnews.com⟩.
[15]David Halliday, Robert Resnick, and Jearl Walker, *Fundamentals of Physics—Extended* (Wiley, New York, 2001), p. 57.
[16]Jerry B. Marion and Stephen T. Thornton, *Classical Dynamics of Particles and Systems* (Harcourt, New York, 1995), p. 59.
[17]Chester R. Kyle, ''Selecting cycling equipment'' in *High-Tech Cycling*, 2nd ed., edited by Edmund R. Burke (Human Kinetics, Champaign, IL, 2003), pp. 1–48.
[18]Eugene Hecht, *Physics*, 2nd ed. (Brooks/Cole, Pacific Grove, CA, 1998), p. 113.
[19]Chester R. Kyle, ''Mechanical factors affecting the speed of a cycle'' in *Science of Cycling*, edited by Edmund R. Burke (Human Kinetics, Champaign, IL, 1986), pp. 123–136.
[20]P. E. DiPrampero, G. Cortili, P. Mognoni, and F. Saibene, ''Equations of motion of a cyclist,'' J. Appl. Physiol.: Respir., Environ. Exercise Physiol. **47**, 201–206 (1979).
[21]Frank Rowland Whitt and David Gordon Wilson, *Bicycling Science* (MIT Press, Cambridge, MA, 1982).
[22]Obviously, if a biker is not outputting enough power on a steep incline, the biker is not exerting enough force. For too small a power in our model, the biker's velocity can switch directions, which means the biker is no longer going up the hill.
[23]Jeffrey P. Broker, ''Cycling power: Road and mountain'' in *High-Tech Cycling*, 2nd ed., edited by Edmund R. Burke (Human Kinetics, Champaign, IL, 2003), pp. 147–174.
[24]A. Lucia, J. Hoyos, and J. L. Chicharro, ''Physiology of professional road racing,'' Med. Sci. Sports Exercise **31**, 325–337 (2001).
[25]Tao Pang, *An Introduction to Computational Physics* (Cambridge University Press, UK, 1997), pp. 5 and 57.
[26]J. Hagberg and S. McCole, ''Energy expenditure during cycling'' in *High-Tech Cycling*, 2nd ed., edited by Edmund R. Burke (Human Kinetics, Champaign, IL, 1996), pp. 167–184.